Paper Reference(s)

6675/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced Level

Wednesday 21 June 2006 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP2), the paper reference (6675), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. Find the values of x for which

$$5 \cosh x - 2 \sinh x = 11,$$

giving your answers as natural logarithms.

(6)

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2. The point S, which lies on the positive x-axis, is a focus of the ellipse with equation $\frac{x^2}{4} + y^2 = 1$.
Given that S is also the focus of a parabola P, with vertex at the origin, find

(a) a cartesian equation for P,

(4)

(b) an equation for the directrix of P.

(1)

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3. The radius of curvature of a curve C, at any point on C, is $e^{\sin\psi}\cos\psi$, where ψ is the angle between the tangent to C at P and the positive axis, and $0 \leq \psi \leq \frac{\pi}{2}$.

Taking $s = 0$ at $\psi = 0$, find an intrinsic equation for C.

(4)

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4. The curve C has equation $y = \arctan x^2$, $0 \leq y < \frac{\pi}{2}$.

Find, in surd form, the value of the radius of curvature of C at the point where $x = 1$.

(6)

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5. The curve with equation

$$y = -x + \tanh 4x, \quad x \geq 0,$$

has a maximum turning point A.

(a) Find, in exact logarithmic form, the x-coordinate of A.

(4)

(b) Show that the y-coordinate of A is $\frac{1}{4}\{2\sqrt{3} - \ln(2 + \sqrt{3})\}$.

(3)

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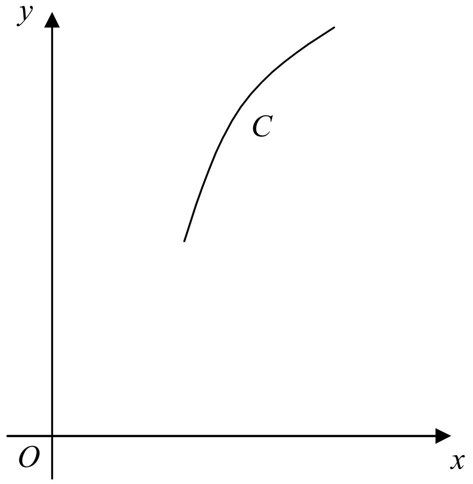
6. **Figure 1**



The curve C, shown in Figure 1, has parametric equations

$$x = t - \ln t,$$
$$y = 4\sqrt{t}, \qquad 1 \leq t \leq 4.$$

(a) Show that the length of C is $3 + \ln 4$.

(7)

The curve is rotated through 2π radians about the x-axis.

(b) Find the exact area of the curved surface generated.

(4)

7. **Figure 2**

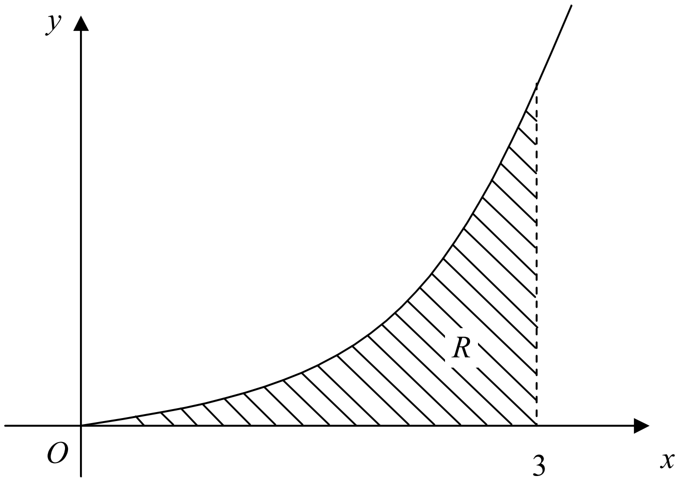


Figure 2 shows a sketch of part of the curve with equation

$$y = x^2 \operatorname{arsinh} x.$$

The region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line $x = 3$.

Show that the area of R is

$$9 \ln (3 + \sqrt{10}) - \tfrac{1}{9}(2 + 7\sqrt{10}).$$

(10)

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8. $$I_n = \int x^n \cosh x \, \mathrm{d}x, \quad n \geq 0.$$

(*a*) Show that, for $n \geq 2$,

$$I_n = x^n \sinh x - nx^{n-1} \cosh x + n(n-1)I_{n-2}.$$

(4)

(*b*) Hence show that

$$I_4 = \mathrm{f}(x) \sinh x + \mathrm{g}(x) \cosh x + C,$$

where $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are functions of x to be found, and C is an arbitrary constant.

(5)

(*c*) Find the exact value of $\int_0^1 x^4 \cosh x \, \mathrm{d}x$, giving your answer in terms of e.

(3)

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9. The ellipse E has equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and the line L has equation $y = mx + c$, where $m > 0$ and $c > 0$.

(a) Show that, if L and E have any points of intersection, the x-coordinates of these points are the roots of the equation

$$(b^2 + a^2m^2)x^2 + 2a^2mcx + a^2(c^2 - b^2) = 0.$$

(2)

Hence, given that L is a tangent to E,

(b) show that $c^2 = b^2 + a^2m^2$.

(2)

The tangent L meets the negative x-axis at the point A and the positive y-axis at the point B, and O is the origin.

(c) Find, in terms of m, a and b, the area of triangle OAB.

(4)

(d) Prove that, as m varies, the minimum area of triangle OAB is ab.

(3)

(e) Find, in terms of a, the x-coordinate of the point of contact of L and E when the area of triangle OAB is a minimum.

(3)

TOTAL FOR PAPER: 75 MARKS

END