June 2006
6689 Decision Maths D1
Mark Scheme

Question Number	Scheme	Marks
1)	eg 52 48 50 45 64 47 53 52 50 48 64 47 53 45 52 50 64 48 53 47 45 52 64 50 53 48 47 45 64 52 53 50 48 47 45 64 53 52 50 48 47 45 no further changes - list sorted	M1 A1 A1✓ A1 [4]

M1 Bubble sort - 1st pass complete - end term 45 or 64, consistent L→R or R→L shuffle, Quicksort M0

A1 First 2 passes correct } condone 'shrinking' list

A1✓ Next 2 passes correct (if L←R next pass) }

A1 Final pass + final statement / rewritten list cso - must see which list

2) (a)	A path from an unmatched vertex in X to an unmatched vertex in Y, which alternately uses arcs in/not in the matching. (where X and Y are distinct sets of vertices.)	B2, 1, 0 (2)
(b)	e.g. $R-B=A-P$ c.s. $R=B-A=P$ $S-F=M-C=D-K$ c.s. $S=F-M=C-D=K$ $\therefore A=P \quad D=K \quad H=Y \quad M=C \quad R=B \quad S=F$	M1 A1 (2) M1 A1 & A1 (3) [7]

(a) B2 A good, complete answer

B1 Partially correct - unmatched to unmatched or arcs in/not in the matching are enough / "bad" sets B1

(b) M1 Path from/to R1s to/from K/P

A1 c.a.o incl c.s.

M1 Second path from remaining LH vertex to remaining RH vertex

A1 c.a.o incl c.s. (Penalise c.s. only once)

A1 Must ✓ from 2 correct paths c a o

Notes for question 1

Q1 Bubble R → L

52 48 50 45 64 47 53 — M1

64 52 48 50 45 53 47

64 53 52 48 50 45 47 — A1

64 53 52 50 48 47 45 — A1

No further change - list sorted — A1

Misread - sorting into ascending order

(note - if candidates reverse list full credit is gained)

L → R (ascending - misread) — (MR)

52 48 50 45 64 47 53 — M1

48 50 45 52 47 53 64

48 45 50 47 52 53 64 — A1

45 48 47 50 52 53 64

45 47 48 50 52 53 64 — A1

No further change - list sorted — A1

(4 - 2 for MR)

R → L

52 48 50 45 64 47 53 — (MR) M1

45 52 48 50 47 64 53

45 47 52 48 50 53 64 — A1

45 47 48 52 50 53 64

45 47 48 50 52 53 64 — A1

No further change - list sorted — A1

(4 - 2 for MR)

Notes for Q 2

(b)(i) $R - B = A - P$

$S - F = M - C = D - K$

(ii) $R - B = A - F = M - C = D - K$

$S - F = A - P$

(iii) $S - F = M - C = D - K$

$R - B = A - P$

(iv) $S - F = M - Y = H - B = A - P$.

$R - B = H - Y = M - C = D - K$

$A = P$

$D = K$

$H = Y$

$M = C$

$R = B$

$S = F$

3)(a)	AC + EG = 44 + 35 = 79 AE + CG = 41 + 36 = 77 * AG + CE = 36 + 45 = 81 Repeat AD, DE, CF and FG	m1 A1 A1 A1√(4)
(b)	length = 394 + 77 = 471 km	B1√(1)
(c)	Since EG is the smallest chance to repeat this hence start and finish at A and C.	m1 A1√(2) [7]

(a) m1 3 pairs of their odd vertices (different)

A1 One pairing and total correct - i.e. one line correct

A1 all 3 pairings and totals correct.

A1√ correct arcs identified - must be 2+ pairings to choose from. AD DG CF EG

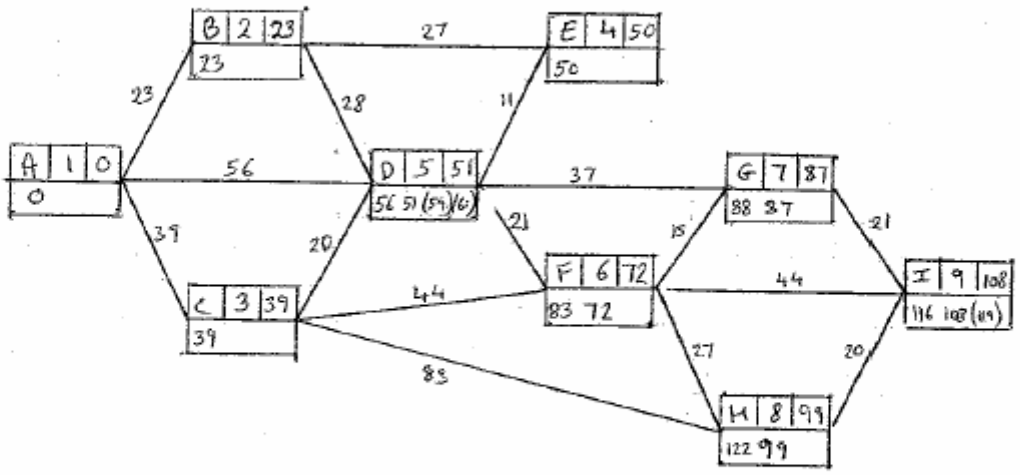
(b) B1 471 (km) 394 + their shortest - must be 2 pairings to choose from.

(c) m1 Identifies {35 EG} as smallest - or identifies their smallest from 2+ pairings + totals

A1√ from 2+ pairings + totals

4(a) A path is a (finite) sequence of edges, such that the end vertex of one edge is the start vertex of the next and in which no vertex appears more than once / no cycles — B2,1,0 (2)

(b)



m1 A1 A1 A1√

Shortest path: A B D F G I , length: 108 miles — A1, A1√ (6)

(c) e.g. 108 − 21 = 87 G I
87 − 15 = 72 F G
72 − 21 = 51 D F
51 − 28 = 23 B D
23 − 23 = 0 A B

or
- trace back from I
- include arc xy if y is already on the path and if the difference in final labels equals the length of arc

B2√√0 (2)

(d) A B E D F G I length 118 miles — m1 A1 (2)

12

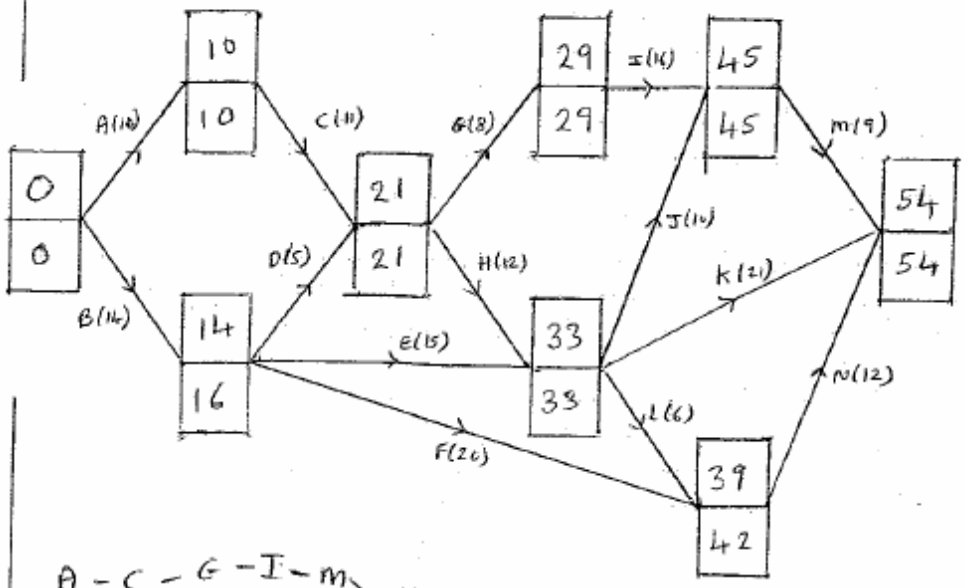
(a) B2 A good, complete description
B1 close – mostly there. "bad" set B1 "route" "series" may be ok

(b) m1 In D, F, G, H or I working values / large replaced by small
A1 A, B, C, E correct labels in a rising sequence
A1 D, F correct labels ✓
A1√ G H I correct labels ✓
} permissible order of labelling once only.
A1 Path c.a.o.
A1√ Length ✓ from I accept 108 if a correct path

(c) B2 ✓ complete version of one of the 2 given explanations
B1 ✓ All there bar one step "bad" set B1 – easy mark

(d) m1 Route A to I including E
A1 c a o

5 (a)



m1
A1 (2)

(b) A – C – G – I – m
H – K

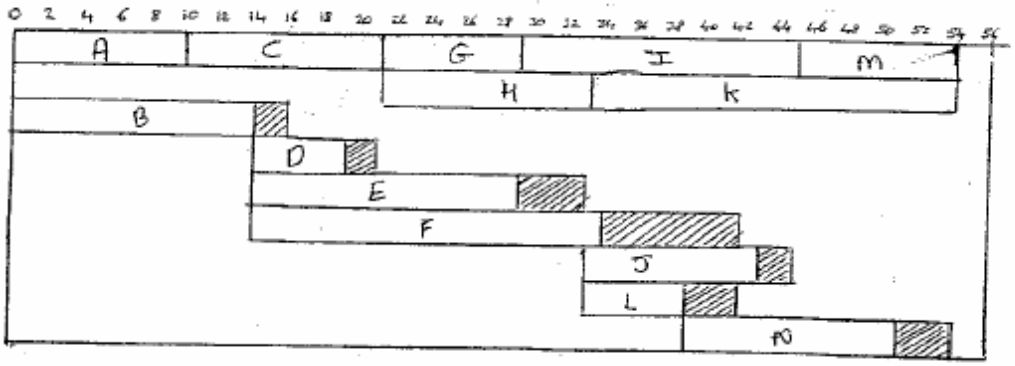
m1
A1 (2)

A1 (1)

(c) Float on D = 21 – 5 – 14 = 2
Float on F = 42 – 20 – 14 = 8

B1✓
m1 A1✓ (3)

(d)



m1
A1
A1✓
A1
(4)

(e) Day 15: C
Day 25: G, H, E, F

B1
B2,1,0 (3)

15

Q5(a) M1 All top boxes completed → increasing generally

A1 C.A.O.

M1 All lower boxes completed ← decreasing generally

A1 c.a.o

(b) A1 c.a.o all 7 listed – no extras

(c) B1√ cao √ from diagram } top ≤ bottom at both ends

M1 method correct or √ correct answer } must see appropriate working for M1

A1√ cao √ from diagram }

(d) M1 At least one of their critical paths + 3 non-critical – stated including floats (must be complete ?)

A1 critical activities correct

A1√ 4 non-critical activities correct √ from diagram must include a float per activity

A1 cao – on non critical

(e) B1 cao

B2 cao

B1 if one extra or one omission

6) (a)

$7x + 10y + 10z + r = 3600$

$6x + 9y + 12z + s = 3600$

$2x + 3y + 4z + t = 2400$

$P - 35x - 55y - 60z = 0$

B2, 1, 0

B2, 0 (4)

(b)

b.v.	x	y	z	r	s	t	value	Row ops.
r	2	5/2	0	1	-5/6	0	600	$R_1 - 10R_2$
z	1/2	3/4	1	0	1/12	0	300	$R_2 \div 12$
t	0	0	0	0	-1/3	1	1200	$R_3 - 4R_2$
P	-5	-10	0	0	5	0	18000	$R_4 + 60R_2$

m1
A1
m1
A1√
B1
(5)

b.v.	x	y	z	r	s	t	value	Row ops
y	4/5	1	0	2/5	-1/3	0	240	$R_1 \div 5/2$
z	-1/10	0	1	-3/10	1/3	0	120	$R_2 - 3/4R_1$
t	0	0	0	0	-1/3	1	1200	R_3 stet
P	3	0	0	4	5/3	0	20400	$R_4 + 10R_1$

m1
A1√
m1
A1
(4)

(c) $P = 20400$ $x = 0$ $y = 240$ $z = 120$

$r = 0$ $s = 0$ $t = 1200$

m1
A2√,1√,0

16

D1 June 2006 Q6(i) - wrong pivot choice

(i) 10 in z column

b.v.	x	y	z	r	s	t	value	Row ops.	mo
z	$\frac{7}{10}$	1	1	$\frac{1}{10}$	0	0	360	$R_1 \div 10$	m1
s	$-\frac{12}{5}$	-3	0	$-\frac{6}{5}$	1	0	-720	$R_2 - 12R_1$	A1
t	$-\frac{4}{5}$	-1	0	$-\frac{2}{5}$	0	1	960	$R_3 - 4R_1$	B0
p	7	5	0	6	0	0	21 600	$R_4 + 60R_1$	m0
									m0

(ii) 4 in z column

b.v.	x	y	z	r	s	t	value	Row ops	mo
r	2	$\frac{5}{2}$	0	1	0	$-\frac{5}{2}$	-2400	$R_1 - 10R_3$	m1
s	0	0	0	0	1	-3	-360	$R_2 - 12R_3$	A1
z	$\frac{1}{2}$	$\frac{3}{4}$	1	0	0	$\frac{1}{4}$	600	$R_3 \div 4$	B0
p	-5	-10	0	0	0	15	36000	$R_4 + 60R_3$	m0

D1 June 2006 Q6(b) Misread

(i) chooses 7 in x column

b.v.	x	y	z	r	s	t	value	Row ops.	
x	1	$\frac{10}{7}$	$\frac{10}{7}$	$\frac{1}{7}$	0	0	$514\frac{2}{7}$	$R_1 \div 7$	$\frac{360}{49}$
s	0	$\frac{3}{7}$	(24/7)	$-\frac{6}{7}$	1	0	$514\frac{2}{7}$	$R_2 - 6R_1$	$\frac{150}{49}$
t	0	$\frac{1}{7}$	$\frac{8}{7}$	$-\frac{2}{7}$	0	1	$1371\frac{3}{7}$	$R_3 - 2R_1$	$\frac{1200}{49}$
P	0	-5	-10	5	0	0	18000	$R_4 + 35R_1$	

1st 5 marks as scheme

b.v.	x	y	z	r	s	t	value	Row ops.		
x	1	(5/4)	0	$\frac{1}{2}$	$-\frac{5}{12}$	0	509.9125...	$R_1 - \frac{10}{7}R_2$	407.93	
z	0	$\frac{1}{8}$	1	$-\frac{1}{4}$	$\frac{7}{24}$	0	150	$R_2 \div \frac{24}{7}$	1200	m1
t	0	0	0	0	$-\frac{1}{3}$	1	1367.930...	$R_3 - \frac{8}{7}R_2$		A1
P	0	$-\frac{15}{4}$	0	$\frac{5}{2}$	$\frac{35}{12}$	0	18030.612...	$R_4 + 10R_2$		

to my final tableau

m1
A1

-2 for misread

(ii) chooses 10 in y column

b.v.	x	y	z	r	s	t	value	Row ops.
y	$\frac{7}{10}$	1	1	$\frac{1}{10}$	0	0	360	$R_1 \div 10$
s	$-\frac{3}{10}$	0	(3)	$-\frac{9}{10}$	1	0	360	$R_2 - 9R_1$
t	$-\frac{1}{10}$	0	1	$-\frac{3}{10}$	0	1	1320	$R_3 - 3R_1$
P	$\frac{7}{2}$	0	-5	$5\frac{1}{2}$	0	0	19800	$R_4 + 55R_1$

1st 5 marks as scheme

to my final tableau

Next 4 marks as scheme

-2 for misread

6 (a) B2 } First 3 equations c.a.o −1 each error, but penalise only 1 error per equation
B1 } inequalities get B0

B2 c a o (B1 for a 'with s_1')

(b) M1 Correct pivot chosen (and some attempt to deal with whole row)

A1 pivot row correct c.a.o including b.v.

M1 Correct row operations used (all 3) - at least 1 non-zero or 1 term correct in each row. Whole row ✓ ⇒ M0

A1✓ non-pivotal row correct; ✓ on error in pivot choice only

(5) B1 Row operations correctly stated (condone lack of $R_2 \div 12$) must be in terms of new pivot row

M1✓ Correct pivot chosen, ✓ from previous tableau. No negatives in value of previous tableau or M0 (+ some attempt to deal with whole row)

A1✓ c a o including b.v. but ✓ from previous tableau

M1 Correct row operations used (all 3) - at least 1 non-zero or 1 term correct in each row. Whole row ✓ ⇒ M0

(4) A1 c.a.o.

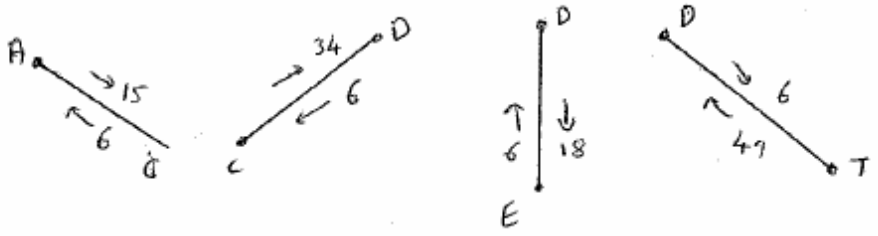
(c) M1 3 variables stated - must have completed b.v. and value columns (or 1's and zeros) on tableau. If reading top → bottom M0. Must be a final tableau - irrespective of M2. Any negative M0

A2✓ all 7 correct

A1✓ at least 4 correct

7) (a) $C_1 = 103$, $C_2 = 177$, flow = 76 — B1, B1, B1 (3)

(b)

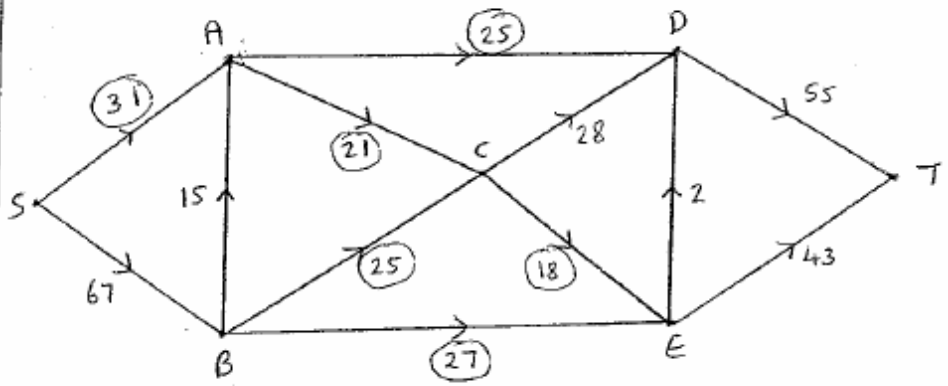


m1
A1 (2)

(c) e.g. S B C D T − 6
S B C D E T − 1
S B A C D E T − 15
max. flow is 98

m1
A3, 2, 1, 0
B1 (5)

(d)



m1
A1 (2)

(e) maximum flow = minimum cut
cut through A D, A C, B C and B E

m1
A1 (2)

14

7)(a) B1 103 cao

B1 177 cao

B1 76 cao

(b) M1 2 numbers added to each of the 4 arcs

A1 cao

(c) M1 1 correct route + flow found (flow > 15 set M0) (condone initial flow augmenting route only if clearly separated from the rest)

A3 all routes + flows found to 22 more

A2 2 routes + flows found to 12+

A1 1 route + flow found & 6+

B1 98 cao

(d) M1 Consistent flow of 77+, complete, clear (doesn't need to √ from (c))

A1 c.a.o

(e) M1 Flow of 98 + cut attempted + max flow min cut theorem referred to (3 out of 4)

A1 cao