Paper Reference(s)

6674/01

Edexcel GCE

Pure Mathematics P4
Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Wednesday 25 January 2006 – Morning
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Lilac)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P4/Further Pure Mathematics FP1), the paper reference (6674), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N20906A

1. Prove that $\sum_{r=1}^{n}(r-1)(r+2) = \frac{1}{3}(n-1)n(n+4)$.

(6)

---

2. Find the set of values of x for which

$$\frac{x^2}{x-2} > 2x.$$

(6)

---

3. Given that $\frac{z+2\text{i}}{z-\lambda\text{i}} = \text{i}$, where λ is a positive, real constant,

(*a*) show that $z = \left(\frac{\lambda}{2}+1\right) + \text{i}\left(\frac{\lambda}{2}-1\right)$.

(5)

Given also that $\arg z = \arctan \frac{1}{2}$, calculate

(*b*) the value of λ,

(3)

(*c*) the value of $|z|^2$.

(2)

---

4. (*a*) Find the general solution of the differential equation

$$\frac{\text{d}^2x}{\text{d}t^2} + 2\frac{\text{d}x}{\text{d}t} + 5x = 0.$$

(4)

(*b*) Given that $x = 1$ and $\frac{\text{d}x}{\text{d}t} = 1$ at $t = 0$, find the particular solution of the differential equation, giving your answer in the form $x = \text{f}(t)$.

(5)

(*c*) Sketch the curve with equation $x = \text{f}(t)$, $0 \leq t \leq \pi$, showing the coordinates, as multiples of π, of the points where the curve cuts the t-axis.

(4)

---

5. The temperature θ °C of a room t hours after a heating system has been turned on is given by

$$\theta = t + 26 - 20e^{-0.5t}, \quad t \geq 0.$$

The heating system switches off when $\theta = 20$. The time $t = \alpha$, when the heating system switches off, is the solution of the equation $\theta - 20 = 0$, where α lies in the interval [1.8, 2].

(*a*) Using the end points of the interval [1.8, 2], find, by linear interpolation, an approximation to α. Give your answer to 2 decimal places.

(4)

(*b*) Taking 1.9 as a first approximation to α, use the Newton-Raphson procedure once to obtain a second approximation to α. Give your answer to 3 decimal places.

(6)

(*c*) Use your answer to part (*b*) to find, to the nearest minute, the time for which the heating system was on.

(1)

---

6. (*a*) Show that the substitution $y = vx$ transforms the differential equation

$$\frac{dy}{dx} = \frac{3x - 4y}{4x + 3y} \qquad \text{(I)}$$

into the differential equation

$$x\frac{dv}{dx} = -\frac{3v^2 + 8v - 3}{3v + 4} \qquad \text{(II).}$$

(4)

(*b*) By solving differential equation (II), find a general solution of differential equation (I).

(5)

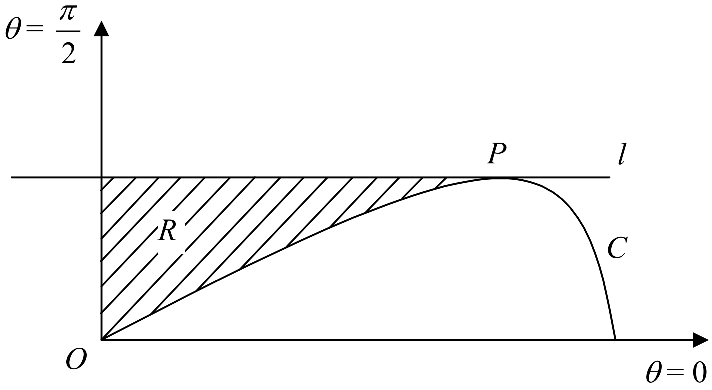
(*c*) Given that $y = 7$ at $x = 1$, show that the particular solution of differential equation (I) can be written as

$$(3y - x)(y + 3x) = 200.$$

(5)

7.

Figure 1



A curve C has polar equation $r^2 = a^2 \cos 2\theta$, $0 \leq \theta \leq \frac{\pi}{4}$. The line l is parallel to the initial line, and l is the tangent to C at the point P, as shown in Figure 1.

(*a*) (i) Show that, for any point on C, $r^2 \sin^2 \theta$ can be expressed in terms of $\sin \theta$ and a only.

(1)

(ii) Hence, using differentiation, show that the polar coordinates of P are $\left(\frac{a}{\sqrt{2}}, \frac{\pi}{6}\right)$.

(6)

The shaded region R, shown in Figure 1, is bounded by C, the line l and the half-line with equation $\theta = \frac{\pi}{2}$.

(*b*) Show that the area of R is $\frac{a^2}{16}(3\sqrt{3} - 4)$.

(8)

TOTAL FOR PAPER: 75 MARKS

END