Paper Reference(s)

6665

Edexcel GCE

Core Mathematics C3

Advanced Level

Monday 23 January 2006 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N23495A

1. **Figure 1**

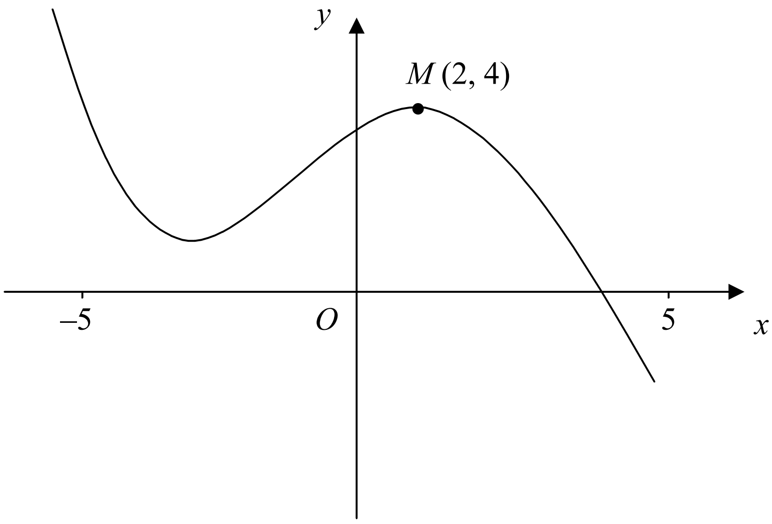

Figure 1 shows the graph of $y = \mathrm{f}(x)$, $-5 \le x \le 5$.

The point M (2, 4) is the maximum turning point of the graph.

Sketch, on separate diagrams, the graphs of

(*a*) $y = \mathrm{f}(x) + 3$,

(2)

(*b*) $y = |\mathrm{f}(x)|$,

(2)

(*c*) $y = \mathrm{f}(|x|)$.

(3)

Show on each graph the coordinates of any maximum turning points.

---

2. Express

$$\frac{2x^2 + 3x}{(2x+3)(x-2)} - \frac{6}{x^2 - x - 2}$$

as a single fraction in its simplest form.

(7)

---

3. The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The x-coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form $y = ax + b$, where a and b are constants.

(5)

---

4. (*a*) Differentiate with respect to x

(i) $x^2 e^{3x+2}$,

(4)

(ii) $\dfrac{\cos(2x^3)}{3x}$.

(4)

(*b*) Given that $x = 4\sin(2y + 6)$, find $\dfrac{dy}{dx}$ in terms of x.

(5)

---

5. $$f(x) = 2x^3 - x - 4.$$

(*a*) Show that the equation f(x) = 0 can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}.$$

(3)

The equation $2x^3 - x - 4 = 0$ has a root between 1.35 and 1.4.

(*b*) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{1}{2}\right)},$$

with $x_0 = 1.35$, to find, to 2 decimal places, the value of x_1, x_2 and x_3.

(3)

The only real root of f(x) = 0 is α.

(*c*) By choosing a suitable interval, prove that $\alpha = 1.392$, to 3 decimal places.

(3)

---

6. $$f(x) = 12 \cos x - 4 \sin x.$$

Given that $f(x) = R \cos (x + \alpha)$, where $R \geq 0$ and $0 \leq \alpha \leq 90°$,

(*a*) find the value of R and the value of α.

(4)

(*b*) Hence solve the equation

$$12 \cos x - 4 \sin x = 7$$

for $0 \leq x < 360°$, giving your answers to one decimal place.

(5)

(*c*) (i) Write down the minimum value of $12 \cos x - 4 \sin x$.

(1)

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs.

(2)

---

7. (*a*) Show that

(i) $\dfrac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x, \quad x \neq (n - \tfrac{1}{4})\pi,\ n \in \mathbb{Z},$

(2)

(ii) $\tfrac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \tfrac{1}{2}$.

(3)

(*b*) Hence, or otherwise, show that the equation

$$\cos \theta \left(\frac{\cos 2\theta}{\cos \theta + \sin \theta} \right) = \frac{1}{2}$$

can be written as

$$\sin 2\theta = \cos 2\theta.$$

(3)

(*c*) Solve, for $0 \leq \theta < 2\pi$,

$$\sin 2\theta = \cos 2\theta,$$

giving your answers in terms of π.

(4)

---

8. The functions f and g are defined by

$$\text{f}: x \mapsto 2x + \ln 2, \qquad x \in \mathbb{R},$$

$$\text{g}: x \mapsto \text{e}^{2x}, \qquad x \in \mathbb{R}.$$

(*a*) Prove that the composite function gf is

$$\text{gf}: x \mapsto 4\text{e}^{4x}, \qquad x \in \mathbb{R}.$$

(4)

(*b*) Sketch the curve with equation $y = \text{gf}(x)$, and show the coordinates of the point where the curve cuts the y-axis.

(1)

(*c*) Write down the range of gf.

(1)

(*d*) Find the value of x for which $\frac{\text{d}}{\text{d}x}[\text{gf}(x)] = 3$, giving your answer to 3 significant figures.

(4)

TOTAL FOR PAPER: 75 MARKS

END