GCE

Edexcel GCE

Core Mathematics C4 (6666)

January 2006

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Mark Scheme (Results)

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<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>1.</td><td>Differentiates</td><td>M1</td></tr>
<tr><td></td><td>to obtain : $6x+8y\frac{dy}{dx}-2,$</td><td>A1,</td></tr>
<tr><td></td><td>$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+(6x\frac{dy}{dx}+6y)=0$</td><td>+(B1)</td></tr>
<tr><td></td><td>$\left[\frac{dy}{dx}=\frac{2-6x-6y}{6x+8y}\right]$</td><td></td></tr>
<tr><td></td><td>Substitutes $x = 1, y = -2$ into expression involving $\frac{dy}{dx}$, to give $\frac{dy}{dx}=-\frac{8}{10}$</td><td>M1, A1</td></tr>
<tr><td></td><td>Uses line equation with numerical ‘gradient’ $y-(-2)=(\text{their gradient})(x-1)$
or finds c and uses $y=(\textit{their gradient})\ x+"c"$</td><td>M1</td></tr>
<tr><td></td><td>To give $5y+4x+6=0$ (or equivalent = 0)</td><td>A1√ [7]</td></tr>
<tr><td>2.
(a)</td><td>
<table>
<tr><td>x</td><td>0</td><td>$\frac{\pi}{16}$</td><td>$\frac{\pi}{8}$</td><td>$\frac{3\pi}{16}$</td><td>$\frac{\pi}{4}$</td></tr>
<tr><td>y</td><td>1</td><td>1.01959</td><td>1.08239</td><td>1.20269</td><td>1.41421</td></tr>
</table>
</td><td>M1 A1</td></tr>
<tr><td></td><td>M1 for one correct, A1 for all correct</td><td>(2)</td></tr>
<tr><td>(b)</td><td>Integral = $\frac{1}{2}\times\frac{\pi}{16}\times\{1+1.4142+2(1.01959+\ldots+1.20269)\}$</td><td>M1 A1√</td></tr>
<tr><td></td><td>$\left(=\frac{\pi}{32}\ x\ 9.02355\right)$ = 0.8859</td><td>A1 cao (3)</td></tr>
<tr><td>(c)</td><td>Percentage error = $\frac{approx-0.88137}{0.88137}\times 100$ = 0.51 % (allow 0.5% to 0.54% for A1)</td><td>M1 A1 (2)</td></tr>
<tr><td></td><td>M1 gained for $(\pm)\ \frac{approx-\ln(1+\sqrt{2})}{\ln(1+\sqrt{2})}$</td><td>[7]</td></tr>
</table>

Question Number	Scheme	Marks
3.	Uses substitution to obtain $x = \mathrm{f}(u)$ $\left[\frac{u^2+1}{2}\right]$,	M1
	and to obtain $u\frac{du}{dx} =$ const. or equiv.	M1
	Reaches $\int \frac{3(u^2+1)}{2u} u du$ or equivalent	A1
	Simplifies integrand to $\int\left(3u^2+\frac{3}{2}\right)\mathrm{d}u$ or equiv.	M1
	Integrates to $\frac{1}{2}u^3+\frac{3}{2}u$	M1 A1√
	A1√ dependent on all previous Ms	
	Uses new limits 3 and 1 substituting and subtracting (or returning to function of x with old limits)	M1
	To give 16 cso	A1 **[8]**
	"By Parts"	
	Attempt at " right direction" by parts M1	
	[$3x\left(2x-1)^{\frac{1}{2}}\right)$ – $\{\int 3\left(2x-1\right)^{\frac{1}{2}}\,\mathrm{d}x\}$] M1{M1A1}	
	……………. – $\left(2x-1\right)^{\frac{3}{2}}$ M1A1√	
	Uses limits 5 and 1 correctly; [42 – 26] 16 M1A1	

4.	Attempts $V = \pi\int x^2e^{2x}\mathrm{d}x$	M1
	$=\pi\left[\frac{x^2e^{2x}}{2}-\int xe^{2x}dx\right]$ (M1 needs parts in the correct direction)	M1 A1
	$= \pi\left[\frac{x^2e^{2x}}{2}-\left(\frac{xe^{2x}}{2}-\int\frac{e^{2x}}{2}dx\right)\right]$ (M1 needs second application of parts)	M1 A1√
	M1A1√ refers to candidates $\int xe^{2x}\,\mathrm{d}x$, but dependent on prev. M1	
	$= \pi\left[\frac{x^2e^{2x}}{2}-\left(\frac{xe^{2x}}{2}-\frac{e^{2x}}{4}\right)\right]$	A1 cao
	Substitutes limits 3 and 1 and subtracts to give… [dep. on second and third Ms]	dM1
	$=\pi\left[\frac{13}{4}e^6-\frac{1}{4}e^2\right]$ or any correct exact equivalent.	A1 **[8]**
	[Omission of π loses first and last marks only]	

Question Number	Scheme	Marks
5. (a)	Considers $3x^2+16=A(2+x)^2+B(1-3x)(2+x)+C(1-3x)$	
	and substitutes x = –2 , or x = 1/3 ,	M1
	or compares coefficients and solves simultaneous equations	
	To obtain A = 3, and C = 4	A1, A1
	Compares coefficients or uses simultaneous equation to show B = 0.	B1 (4)
(b)	Writes $3(1-3x)^{-1}+4(2+x)^{-2}$	M1
	$=3(1+3x,+9x^2+27x^3+......)$ +	(M1, A1)
	$\frac{4}{4}(1+\frac{(-2)}{1}\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{1.2}\left(\frac{x}{2}\right)^2+\frac{(-2)(-3)(-4)}{1.2.3}\left(\frac{x}{2}\right)^3+....)$	(M1 A1)
	$=4+8x,\ +27\tfrac{3}{4}x^2+80\tfrac{1}{2}x^3+...$	A1, A1 (7)
	Or uses $(3x^2+16)(1-3x)^{-1}(2+x)^{-2}$	M1
	$(3x^2+16)\ (1+3x,+9x^2+27x^3+)\ \times$	(M1A1)×
	$\frac{1}{4}(1+\frac{(-2)}{1}\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{1.2}\left(\frac{x}{2}\right)^2+\frac{(-2)(-3)(-4)}{1.2.3}\left(\frac{x}{2}\right)^3)$	(M1A1)
	$=4+8x,+27\tfrac{3}{4}x^2+80\tfrac{1}{2}x^3+...$	A1, A1 (7)
		[11]

6. (a)	$\lambda=-4\to a=18,$ $\mu=1\to b=9$	M1 A1, A1 (3)
(b)	$\begin{pmatrix}8+\lambda\\12+\lambda\\14-\lambda\end{pmatrix}\bullet\begin{pmatrix}1\\1\\-1\end{pmatrix}=0$	M1
	$\therefore 8+\lambda+12+\lambda-14+\lambda=0$	A1
	Solves to obtain λ ($\lambda=-2$)	dM1
	Then substitutes value for λ to give P at the point (6, 10, 16) (any form)	M1, A1 (5)
(c)	OP = $\sqrt{36+100+256}$	M1
	(= $\sqrt{392}$) $=14\sqrt{2}$	A1 cao (2) **[10]**
7. (a)	$\frac{dV}{dr}=4\pi r^2$	B1 (1)
(b)	Uses $\frac{dr}{dt}=\frac{dV}{dt}.\frac{dr}{dV}$ in any form, $=\frac{1000}{4\pi r^2(2t+1)^2}$	M1,A1 (2)
(c)	$V=\int 1000(2t+1)^{-2}dt$ and integrate to $p\,(2t+1)^{-1}$, $=-500(2t+1)^{-1}(+c)$	M1, A1
	Using V=0 when t=0 to find c , (c = 500 , or equivalent)	M1
	$\therefore V=500(1-\frac{1}{2t+1})$ (any form)	A1 (4)
(d)	(i) Substitute t = 5 to give V,	M1,
	then use $r=\sqrt[3]{\left(\frac{3V}{4\pi}\right)}$ to give *r* , = *4.77*	M1, A1 (3)
	(ii) Substitutes t = 5 and r = ‘their value’ into ‘their’ part (b)	M1
	$\frac{\mathrm{d}r}{\mathrm{d}t}=0.0289$ $(\approx 2.90x10^{-2})$ (cm/s) $*$ AG	A1 (2) **[12]**

8. (a)	Solves y = 0 ⇒ $\cos t = \frac{1}{2}$ to obtain $t = \frac{\pi}{3}\ or\ \frac{5\pi}{3}$ (need both for A1)	M1 A1
	Or substitutes **both** values of *t* and shows that $y = 0$	(2)
(b)	$\frac{dx}{dt} = 1 - 2\cos t$	M1 A1
	Area= $\int y dx = \int_{\pi/3}^{5\pi/3} (1-2\cos t)(1-2\cos t)dt$ = $\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1-2\cos t)^2 dt$ * AG	B1
		(3)
(c)	Area = $\int 1 - 4\cos t + 4\cos^2 t dt$ 3 terms	M1
	= $\int 1 - 4\cos t + 2(\cos 2t + 1)dt$ (use of correct double angle formula)	M1
	= $\int 3 - 4\cos t + 2\cos 2t dt$	
	= $\left[3t - 4\sin t + \sin 2t\right]$	M1 A1
	Substitutes the two correct limits $t = \frac{5\pi}{3}$ and $\frac{\pi}{3}$ and subtracts.	M1
	$= 4\pi + 3\sqrt{3}$	A1A1
		(7)
		[12]