Paper Reference(s)

6666

Edexcel GCE

Core Mathematics C4

Advanced Level

Monday 23 January 2006 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N23553A

1. A curve C is described by the equation

$$3x^2 + 4y^2 - 2x + 6xy - 5 = 0.$$

Find an equation of the tangent to C at the point $(1, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

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2. (*a*) Given that $y = \sec x$, complete the table with the values of y corresponding to $x = \frac{\pi}{16}, \frac{\pi}{8}$ and $\frac{\pi}{4}$.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	1			1.20269	

(2)

(*b*) Use the trapezium rule, with all the values for y in the completed table, to obtain an estimate for $\int_0^{\frac{\pi}{4}} \sec x \, \mathrm{d}x$. Show all the steps of your working and give your answer to 4 decimal places.

(3)

The exact value of $\int_0^{\frac{\pi}{4}} \sec x \, \mathrm{d}x$ is $\ln(1 + \sqrt{2})$.

(*c*) Calculate the % error in using the estimate you obtained in part (*b*).

(2)

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3. Using the substitution $u^2 = 2x - 1$, or otherwise, find the exact value of

$$\int_1^5 \frac{3x}{\sqrt{(2x-1)}} \, \mathrm{d}x.$$

(8)

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4. **Figure 1**

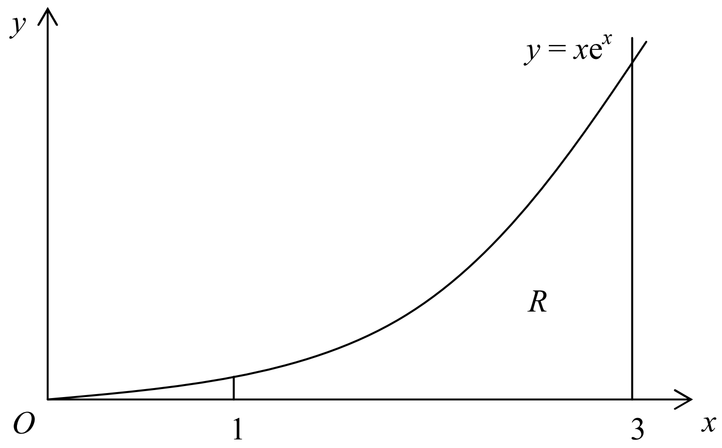

Figure 1 shows the finite region R, which is bounded by the curve $y = xe^x$, the line $x = 1$, the line $x = 3$ and the x-axis.

The region R is rotated through 360 degrees about the x-axis.

Use integration by parts to find an exact value for the **volume** of the solid generated.

(8)

5.

$$f(x) = \frac{3x^2 + 16}{(1-3x)(2+x)^2} = \frac{A}{(1-3x)} + \frac{B}{(2+x)} + \frac{C}{(2+x)^2}, \quad |x| < \tfrac{1}{3}.$$

(*a*) Find the values of A and C and show that $B = 0$.

(4)

(*b*) Hence, or otherwise, find the series expansion of $f(x)$, in ascending powers of x, up to and including the term in x^3. Simplify each term.

(7)

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6. The line l_1 has vector equation

$$\mathbf{r} = 8\mathbf{i} + 12\mathbf{j} + 14\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

where λ is a parameter.

The point A has coordinates $(4, 8, a)$, where a is a constant. The point B has coordinates $(b, 13, 13)$, where b is a constant. Points A and B lie on the line l_1.

(*a*) Find the values of a and b.

(3)

Given that the point O is the origin, and that the point P lies on l_1 such that OP is perpendicular to l_1,

(*b*) find the coordinates of P.

(5)

(*b*) Hence find the distance OP, giving your answer as a simplified surd.

(2)

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7. The volume of a spherical balloon of radius r cm is V cm^3, where $V = \frac{4}{3}\pi r^3$.

(*a*) Find $\dfrac{\mathrm{d}V}{\mathrm{d}r}$.

(1)

The volume of the balloon increases with time t seconds according to the formula

$$\frac{\mathrm{d}V}{\mathrm{d}r} = \frac{1000}{(2t+1)^2}, \quad t \geq 0.$$

(*b*) Using the chain rule, or otherwise, find an expression in terms of r and t for $\dfrac{\mathrm{d}r}{\mathrm{d}t}$.

(2)

(*c*) Given that $V = 0$ when $t = 0$, solve the differential equation $\dfrac{\mathrm{d}V}{\mathrm{d}r} = \dfrac{1000}{(2t+1)^2}$, to obtain V in terms of t.

(4)

(*d*) Hence, at time $t = 5$,

(i) find the radius of the balloon, giving your answer to 3 significant figures,

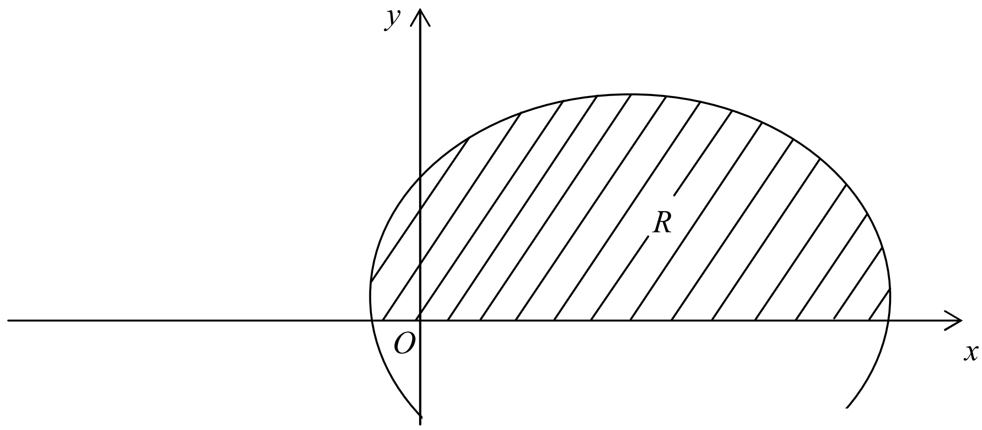
(3)

(ii) show that the rate of increase of the radius of the balloon is approximately 2.90×10^{-2} cm s^{-1}.

(2)

8.

Figure 2



The curve shown in Figure 2 has parametric equations

$$x = t - 2\sin t, \qquad y = 1 - \cos t, \qquad 0 \le t \le 2\pi.$$

(*a*) Show that the curve crosses the x-axis where $t = \frac{\pi}{3}$ and $t = \frac{5\pi}{3}$.

(2)

The finite region R is enclosed by the curve and the x-axis, as shown shaded in Figure 2.

(*b*) Show that the area R is given by the integral

$$\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)^2 \quad \mathrm{d}t \,.$$

(3)

(*c*) Use this integral to find the exact value of the shaded area.

(7)

TOTAL FOR PAPER: 75 MARKS

END