

Question Number	Scheme	Marks												
1.	<p>Differentiates</p> <p>to obtain : <math>6x + 8y \frac{dy}{dx} - 2,</math>  <math>\dots\dots\dots + (6x \frac{dy}{dx} + 6y) = 0</math></p> $\left[ \frac{dy}{dx} = \frac{2 - 6x - 6y}{6x + 8y} \right]$ <p>Substitutes <math>x = 1, y = -2</math> into expression involving <math>\frac{dy}{dx}</math>, to give <math>\frac{dy}{dx} = -\frac{8}{10}</math></p> <p>Uses line equation with numerical 'gradient' <math>y - (-2) = (\text{their gradient})(x - 1)</math>  or finds <math>c</math> and uses <math>y = (\text{their gradient})x + "c"</math></p> <p>To give <math>5y + 4x + 6 = 0</math> (or equivalent = 0)</p>	<p>M1 A1, +(B1)</p> <p>M1, A1</p> <p>M1</p> <p>A1√ <b>[7]</b></p>												
2. (a)	<table border="1" data-bbox="225 976 1294 1133"> <tr> <td><math>x</math></td> <td>0</td> <td><math>\frac{\pi}{16}</math></td> <td><math>\frac{\pi}{8}</math></td> <td><math>\frac{3\pi}{16}</math></td> <td><math>\frac{\pi}{4}</math></td> </tr> <tr> <td><math>y</math></td> <td>1</td> <td><b>1.01959</b></td> <td><b>1.08239</b></td> <td>1.20269</td> <td><b>1.41421</b></td> </tr> </table> <p>M1 for one correct, A1 for all correct</p>	$x$	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$	$y$	1	<b>1.01959</b>	<b>1.08239</b>	1.20269	<b>1.41421</b>	<p>M1 A1 (2)</p>
$x$	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$									
$y$	1	<b>1.01959</b>	<b>1.08239</b>	1.20269	<b>1.41421</b>									
(b)	<p>Integral = <math>\frac{1}{2} \times \frac{\pi}{16} \times \{1 + 1.4142 + 2(1.01959 + \dots + 1.20269)\}</math>  <math>\left( = \frac{\pi}{32} \times 9.02355 \right) = 0.8859</math></p>	<p>M1 A1√ A1 cao (3)</p>												
(c)	<p>Percentage error = <math>\frac{\text{approx} - 0.88137}{0.88137} \times 100 = 0.51\%</math> (allow 0.5% to 0.54% for A1)</p> <p>M1 gained for <math>(\pm) \frac{\text{approx} - \ln(1 + \sqrt{2})}{\ln(1 + \sqrt{2})}</math></p>	<p>M1 A1 (2) <b>[7]</b></p>												

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3.	<p>Uses substitution to obtain <math>x = f(u) \left[ \frac{u^2 + 1}{2} \right]</math>,</p> <p>and to obtain <math>u \frac{du}{dx} = \text{const. or equiv.}</math></p> <p>Reaches <math>\int \frac{3(u^2 + 1)}{2u} u du</math> or equivalent</p> <p>Simplifies integrand to <math>\int \left( 3u^2 + \frac{3}{2} \right) du</math> or equiv.</p> <p>Integrates to <math>\frac{1}{2}u^3 + \frac{3}{2}u</math></p> <p>A1√ dependent on all previous Ms</p> <p>Uses new limits 3 and 1 substituting and subtracting (or returning to function of x with old limits)</p> <p>To give 16 cso</p> <hr/> <p>“By Parts”                      Attempt at “right direction” by parts M1  <math display="block">\left[ 3x \left( 2x - 1 \right)^{\frac{1}{2}} \right] - \left\{ \int 3 \left( 2x - 1 \right)^{\frac{1}{2}} dx \right\}</math> M1{M1A1}  <math display="block">\dots\dots\dots - \left( 2x - 1 \right)^{\frac{3}{2}}</math> M1A1√                      Uses limits 5 and 1 correctly; [42 – 26] 16 M1A1</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1 A1√</p> <p>M1</p> <p>A1</p> <p><b>[8]</b></p>

4.	<p>Attempts <math>V = \pi \int x^2 e^{2x} dx</math></p> $= \pi \left[ \frac{x^2 e^{2x}}{2} - \int x e^{2x} dx \right]$ <p>(M1 needs parts in the correct direction)</p> $= \pi \left[ \frac{x^2 e^{2x}}{2} - \left( \frac{x e^{2x}}{2} - \int \frac{e^{2x}}{2} dx \right) \right]$ <p>(M1 needs second application of parts)</p> <p>M1A1√ refers to candidates <math>\int x e^{2x} dx</math>, but dependent on prev. M1</p> $= \pi \left[ \frac{x^2 e^{2x}}{2} - \left( \frac{x e^{2x}}{2} - \frac{e^{2x}}{4} \right) \right]$ <p>Substitutes limits 3 and 1 and subtracts to give... [dep. on second and third Ms]</p> $= \pi \left[ \frac{13}{4} e^6 - \frac{1}{4} e^2 \right] \text{ or any correct exact equivalent.}$ <p>[Omission of <math>\pi</math> loses first and last marks only]</p>	<p>M1</p> <p>M1 A1</p> <p>M1 A1√</p> <p>A1 cao</p> <p>dM1</p> <p>A1</p> <p><b>[8]</b></p>
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5. (a)	<p>Considers <math>3x^2 + 16 = A(2+x)^2 + B(1-3x)(2+x) + C(1-3x)</math></p> <p>and substitutes <math>x = -2</math> , or <math>x = 1/3</math> ,</p> <p>or compares coefficients and solves simultaneous equations</p> <p>To obtain <math>A = 3</math>, and <math>C = 4</math></p> <p>Compares coefficients or uses simultaneous equation to show <math>B = 0</math>.</p>	<p>M1</p> <p>A1, A1</p> <p>B1</p> <p>(4)</p>
5. (b)	<p>Writes <math>3(1-3x)^{-1} + 4(2+x)^{-2}</math></p> <p><math>= 3(1+3x, +9x^2 + 27x^3 + \dots) +</math></p> $\frac{4}{4}\left(1 + \frac{(-2)}{1}\left(\frac{x}{2}\right) + \frac{(-2)(-3)}{1.2}\left(\frac{x}{2}\right)^2 + \frac{(-2)(-3)(-4)}{1.2.3}\left(\frac{x}{2}\right)^3 + \dots\right)$ <p><math>= 4 + 8x, + 27\frac{3}{4}x^2 + 80\frac{1}{2}x^3 + \dots</math></p> <p><b>Or</b> uses <math>(3x^2 + 16)(1-3x)^{-1}(2+x)^{-2}</math></p> <p><math>(3x^2 + 16) (1 + 3x, +9x^2 + 27x^3 +) \times</math></p> $\frac{1}{4}\left(1 + \frac{(-2)}{1}\left(\frac{x}{2}\right) + \frac{(-2)(-3)}{1.2}\left(\frac{x}{2}\right)^2 + \frac{(-2)(-3)(-4)}{1.2.3}\left(\frac{x}{2}\right)^3 + \dots\right)$ <p><math>= 4 + 8x, + 27\frac{3}{4}x^2 + 80\frac{1}{2}x^3 + \dots</math></p>	<p>M1</p> <p>(M1, A1)</p> <p>( M1 A1 )</p> <p>A1, A1</p> <p>(7)</p> <p>M1</p> <p>(M1A1)×</p> <p>(M1A1)</p> <p>A1, A1</p> <p>(7)</p> <p>[11]</p>

<p>6. (a)</p> <p>(b)</p> <p>(c)</p>	<p><math>\lambda = -4 \rightarrow a = 18, \quad \mu = 1 \rightarrow b = 9</math></p> $\begin{pmatrix} 8 + \lambda \\ 12 + \lambda \\ 14 - \lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 0$ <p><math>\therefore 8 + \lambda + 12 + \lambda - 14 + \lambda = 0</math></p> <p>Solves to obtain <math>\lambda</math> (<math>\lambda = -2</math>)</p> <p>Then substitutes value for <math>\lambda</math> to give P at the point (6, 10, 16) (any form)</p> <p><math>OP = \sqrt{36 + 100 + 256}</math></p> <p><math>(= \sqrt{392}) = 14\sqrt{2}</math></p>	<p>M1 A1, A1 (3)</p> <p>M1</p> <p>A1</p> <p>dM1</p> <p>M1, A1 (5)</p> <p>M1</p> <p>A1 cao (2)</p> <p><b>[10]</b></p>
<p>7. (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	<p><math>\frac{dV}{dr} = 4\pi r^2</math></p> <p>Uses <math>\frac{dr}{dt} = \frac{dV}{dt} \cdot \frac{dr}{dV}</math> in any form, <math>= \frac{1000}{4\pi r^2 (2t+1)^2}</math></p> <p><math>V = \int 1000(2t+1)^{-2} dt</math> and integrate to <math>p(2t+1)^{-1}, \quad = -500(2t+1)^{-1} (+c)</math></p> <p>Using <math>V=0</math> when <math>t=0</math> to find <math>c</math>, (<math>c = 500</math>, or equivalent)</p> <p><math>\therefore V = 500(1 - \frac{1}{2t+1})</math> (any form)</p> <p>(i) Substitute <math>t = 5</math> to give <math>V</math>, then use <math>r = \sqrt[3]{\left(\frac{3V}{4\pi}\right)}</math> to give <math>r, = 4.77</math></p> <p>(ii) Substitutes <math>t = 5</math> and <math>r =</math> 'their value' into 'their' part (b)</p> <p><math>\frac{dr}{dt} = 0.0289</math> (<math>\approx 2.90 \times 10^{-2}</math>) (cm/s) * AG</p>	<p>B1 (1)</p> <p>M1, A1 (2)</p> <p>M1, A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1,</p> <p>M1, A1 (3)</p> <p>M1</p> <p>A1 (2)</p> <p><b>[12]</b></p>

8.	<p>(a) Solves <math>y = 0 \Rightarrow \cos t = \frac{1}{2}</math> to obtain <math>t = \frac{\pi}{3}</math> or <math>\frac{5\pi}{3}</math> (need both for A1)</p> <p>Or substitutes <b>both</b> values of <math>t</math> and shows that <math>y = 0</math></p> <p>(b)</p> $\frac{dx}{dt} = 1 - 2 \cos t$ $\text{Area} = \int y dx = \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2 \cos t)(1 - 2 \cos t) dt = \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2 \cos t)^2 dt \quad * \quad \text{AG}$ <p>(c)</p> $\begin{aligned} \text{Area} &= \int 1 - 4 \cos t + 4 \cos^2 t dt && \text{3 terms} \\ &= \int 1 - 4 \cos t + 2(\cos 2t + 1) dt && \text{(use of correct double angle formula)} \\ &= \int 3 - 4 \cos t + 2 \cos 2t dt \\ &= [3t - 4 \sin t + \sin 2t] \end{aligned}$ <p>Substitutes the two correct limits <math>t = \frac{5\pi}{3}</math> and <math>\frac{\pi}{3}</math> and subtracts.</p> $= 4\pi + 3\sqrt{3}$	
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