Leave blank

1.

Figure 1

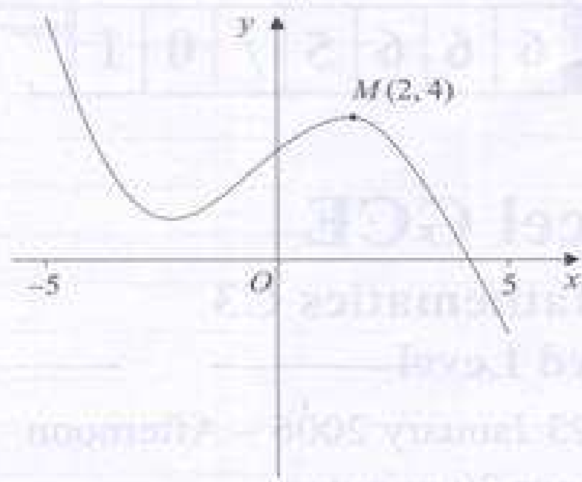

Figure 1 shows the graph of $y = \mathrm{f}(x)$, $-5 \leqslant x \leqslant 5$.
The point $M(2, 4)$ is the maximum turning point of the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = \mathrm{f}(x) + 3$, **(2)**

(b) $y = |\mathrm{f}(x)|$, **(2)**

(c) $y = \mathrm{f}(|x|)$. **(3)**

Show on each graph the coordinates of any maximum turning points.

2. Express

$$\frac{2x^2+3x}{(2x+3)(x-2)}-\frac{6}{x^2-x-2}$$

as a single fraction in its simplest form.

(7)

Leave blank

3. The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The x-coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form $y = ax + b$, where a and b are constants.

(5)

N 2 3 4 9 5 A 0 6 2 0

Leave blank

4. (a) Differentiate with respect to x

(i) $x^2 e^{3x+2}$,

(4)

(ii) $\dfrac{\cos(2x^3)}{3x}$.

(4)

(b) Given that $x = 4\sin(2y + 6)$, find $\dfrac{dy}{dx}$ in terms of x.

(5)

Leave blank

5. $$\mathrm{f}(x) = 2x^3 - x - 4.$$

(a) Show that the equation $\mathrm{f}(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}.$$

(3)

The equation $2x^3 - x - 4 = 0$ has a root between 1.35 and 1.4.

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{1}{2}\right)},$$

with $x_0 = 1.35$, to find, to 2 decimal places, the values of x_1, x_2 and x_3.

(3)

The only real root of $\mathrm{f}(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.392$, to 3 decimal places.

(3)

Leave blank

6.

$$f(x) = 12 \cos x - 4 \sin x.$$

Given that $f(x) = R \cos(x + \alpha)$, where $R \geqslant 0$ and $0 \leqslant \alpha \leqslant 90°$,

(a) find the value of R and the value of α.

(4)

(b) Hence solve the equation

$$12 \cos x - 4 \sin x = 7$$

for $0 \leqslant x < 360°$, giving your answers to one decimal place.

(5)

(c) (i) Write down the minimum value of $12 \cos x - 4 \sin x$.

(1)

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs.

(2)

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7. (a) Show that

(i) $\dfrac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x, \quad x \neq (n - \frac{1}{4})\pi,\ n \in \mathbb{Z},$

(2)

(ii) $\frac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \frac{1}{2}.$

(3)

(b) Hence, or otherwise, show that the equation

$$\cos\theta\left(\frac{\cos 2\theta}{\cos\theta + \sin\theta}\right) = \frac{1}{2}$$

can be written as

$$\sin 2\theta = \cos 2\theta.$$

(3)

(c) Solve, for $0 \leqslant \theta < 2\pi$,

$$\sin 2\theta = \cos 2\theta,$$

giving your answers in terms of π.

(4)

N 2 3 4 9 5 A 0 1 4 2 0

Leave blank

8. The functions f and g are defined by

$$f : x \rightarrow 2x + \ln 2, \quad x \in \mathbb{R},$$

$$g : x \rightarrow e^{2x}, \quad x \in \mathbb{R}.$$

(a) Prove that the composite function gf is

$$gf : x \rightarrow 4e^{4x}, \quad x \in \mathbb{R}.$$

(4)

(b) In the space provided on page 19, sketch the curve with equation $y = gf(x)$, and show the coordinates of the point where the curve cuts the y-axis.

(1)

(c) Write down the range of gf.

(1)

(d) Find the value of x for which $\frac{d}{dx}[gf(x)] = 3$, giving your answer to 3 significant figures.

(4)