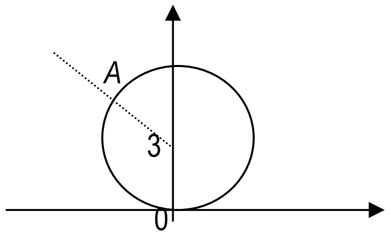
June 2005
6676 Pure P6
Mark Scheme

Question Number	Scheme	Marks
1	(a) $\frac{6x+10}{x+3} = 6 - \frac{8}{x+3}$	B1 (1)
	(b) $u_1 = 5.2 > 5$	B1
	If result true for $n = k$, i.e. $u_k > 5$, $u_{k+1} = 6 - \frac{8}{u_k + 3}$ If $u_k > 5$, then $\frac{8}{u_k+3} < 1$ so $u_{k+1} > 5$	M1A1
	Hence result is true for $n = k + 1$ Conclusion and no wrong working seen	A1 (4)
		[5]
2	(a) (i) $\mathbf{b} \times \mathbf{a}$ is perpendicular to $\mathbf{a}$ (and $\mathbf{b}$)	B1
	$\mathbf{a}.\,\mathbf{b} \times \mathbf{a} = \lvert\mathbf{a}\rvert\,\lvert\mathbf{b} \times \mathbf{a}\rvert \cos 90° = 0$ or equivalent	B1 (2)
	(ii) $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c} \Rightarrow \mathbf{a} \times (\mathbf{b} - \mathbf{c}) = \mathbf{0}$	M1
	As $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{b} \neq \mathbf{c}$,	
	$\mathbf{a}$ is parallel to $(\mathbf{b} - \mathbf{c})$, so $\mathbf{b} - \mathbf{c} = \lambda\,\mathbf{a}$	A1 (2)
	(b) (i) If $\mathbf{A}$ non-singular, then $\mathbf{A}^{-1}\mathbf{AB} = \mathbf{A}^{-1}\mathbf{AC} \Rightarrow \mathbf{B} = \mathbf{C}$ (*)AG	M1A1 (2)
	(ii) $\begin{pmatrix} 3 & 6 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 21 \\ 1 & 7 \end{pmatrix}$	B1
	Set $\begin{pmatrix} 3 & 6 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 21 \\ 1 & 7 \end{pmatrix}$ and finding two equations	M1
	Any non-zero values of a, b, c and d such that $a + 2c = 1$ and $b + 2d = 7$.	A1 (3)
		[9]

Question Number	Scheme	Marks
3	(a) Normal to plane is $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 3 \\ 1 & -2 & 1 \end{vmatrix}$ = 6**i** + **j** – 4**k** (or any multiple)	M1A1 (2)
	(b) Equation of plane is $6x + y - 4z = d$	M1
	Substituting appropriate point in equation to give $6x + y - 4z = 16$ [e.g. (1, 6,–1), (3, –2, 0), (3, 6, 2) etc.]	A1 (2)
	(c) $p = -2$	B1 (1)
	(d) Direction of line is perpendicular to both normals	
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 1 \\ 6 & 1 & -4 \end{vmatrix}$ = –9**i** + 10**j** –11**k**	M1
	[Planes are: $6x + y - 4z = 16$, $x + 2y + z = 2$]	
	Finding a point on line	M1A1
	a and **b** identified	M1
	Any correct equation of correct form e.g. $\left[r - \begin{pmatrix} -3 \\ 6 \\ -7 \end{pmatrix} \right] \text{x} \begin{pmatrix} 9 \\ -10 \\ 11 \end{pmatrix} = 0.$	A1 (5) [10]
	Alternative: Using equations of planes to find general point on line	
	Using equations of planes to form any two of $10x + 9y = 24$, $11x - 9z = 30$, $11y + 10z = -4$ — M1	
	Putting in parametric form — M1	
	e.g. $\left(\lambda, \frac{24 - 10\lambda}{9}, \frac{-30 + 11\lambda}{9} \right)$ — A1	
	a and **b** identified — M1	
	Writing in required form; a correct equation — A1	

4 (a)



Circle — M1

Correct circle. — A1 (2)

(centre (0, 3), radius 3)

(b) Drawing correct **half**-line passing as shown — B1

Find either *x* or *y* coord of *A*. — M1A1

$$z = -\frac{3\sqrt{2}}{2} + \left(3 + \frac{3\sqrt{2}}{2}\right)\text{i}$$ — A1 (4)

[Algebraic approach, i.e. using $y = 3 - x$ and equation of circle will only gain M1A1, unless the second solution is ruled out, when B1 can be given by implication, and final A1, if correct]

(c) $|z - 3\text{i}| = 3 \rightarrow \left|\frac{2\text{i}}{\omega} - 3\text{i}\right| = 3$ — M1

$$\Rightarrow \frac{|2\text{i} - 3\text{i}\omega|}{|\omega|} = 3$$ — A1

$$\Rightarrow |\omega - {}^2/_3| = |\omega|$$ — M1A1

Line with equation $u = {}^1/_3$ $(x = {}^1/_3)$ — A1 (5)

[11]

Some alternatives:

(i) $\omega = \frac{2\text{i}}{x + \text{i}y} = \frac{2\text{i}(x - \text{i}y)}{x^2 + y^2} \Rightarrow u = \frac{2y}{x^2 + y^2},\ v = \frac{2x}{x^2 + y^2}$ M1A1

As $x^2 + y^2 - 6y = 0$, $u = \frac{1}{3}$, M1,A1A1

(ii) $\omega = \frac{2\text{i}}{3\cos\theta + 3\text{i}(1 + \sin\theta)} = \frac{2\text{i}\{\cos\theta - \text{i}(1 + \sin\theta)\}}{3\{\cos^2\theta + (1 + \sin\theta)^2\}}$ M1A1

$= \frac{2}{3}\frac{(1 + \sin\theta) + \text{i}\cos\theta}{2 + 2\sin\theta},\ = \frac{1}{3} + \text{i}\ \frac{\cos\theta}{1 + \sin\theta},$ M1A1

So locus is line $u = \frac{1}{3}$ A1

5	(a) $z^n = e^{in\theta} = (\cos n\theta + i\sin n\theta)$, $z^{-n} = e^{-in\theta} = (\cos n\theta - i\sin n\theta)$	M1
	Completion (needs to be convincing) $z^n - \frac{1}{z^n} = 2i\sin n\theta$ (*)AG	A1 (2)
	(b) $\left(z - \frac{1}{z}\right)^5 = z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$	M1A1
	$= \left(z^5 - \frac{1}{z^5}\right) - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)$	
	$(2i\sin\theta)^5 = 32i\sin^5\theta = 2i\sin 5\theta - 10i\sin 3\theta + 20i\sin\theta$	M1A1
	$\Rightarrow \sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$ (*) AG	A1 (5)
	(c) Finding $\sin^5\theta = ¼\sin\theta$	M1
	$\theta = 0, \pi$ (both)	B1
	$(\sin^4\theta = ¼) \Rightarrow \sin\theta = \pm\frac{1}{\sqrt{2}}$	M1
	$\theta = \frac{\pi}{4}, \frac{3\pi}{4}; \quad \frac{5\pi}{4}, \frac{7\pi}{4}$	A1;A1 (5)
		[12]
6	(a) $\left(\frac{d^2y}{dx^2}\right)_0 = \frac{1}{4}$	B1 (1)
	(b) $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_{-1}}{2h} \Rightarrow \frac{1}{2} \approx \frac{y_1 - y_{-1}}{0.2} \Rightarrow y_1 - y_{-1} \approx 0.1$	M1A1
	$\left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2} \Rightarrow \frac{1}{4} \approx \frac{y_1 - 2 + y_{-1}}{0.01}$	M1
	$\Rightarrow y_1 + y_{-1} \approx 2.0025$	A1
	Adding to give $y_1 \approx 1.05125$	M1A1 (6)
	(c) Diff: $4(1 + x^2)\frac{d^3y}{dx^3} + 8x\frac{d^2y}{dx^2} + 4x\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = \frac{dy}{dx}$	M1A1
	Substituting appropriate vales $\Rightarrow 4\left(\frac{d^3y}{dx^3}\right)_0 = -\frac{3}{2} \Rightarrow \left(\frac{d^3y}{dx^3}\right)_0 = -\frac{3}{8}$	M1A1 (4)
	(d) $y = y_0 + y'_0x + \frac{y''_0}{2!}x^2 + \frac{y'''_0}{3!}x^3 + \ldots = 1 + \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{16}x^3 + \ldots$	M1A1√(2)
	(e) 1.05119	A1 (1)
		[14]

7	(a) Det = –12 –2(2k – 8) + 16 = 20 – 4k (*) AG	M1A1 (2)
	(b) Cofactors $\begin{pmatrix} -4 & 8-2k & 4 \\ 8-2k & 3k-16 & 2 \\ 4 & 2 & -4 \end{pmatrix}$ [A1 each error]	M1A3
	$A^{-1} = \dfrac{1}{20-4k}\begin{pmatrix} -4 & 8-2k & 4 \\ 8-2k & 3k-16 & 2 \\ 4 & 2 & -4 \end{pmatrix}$	M1A1√ (6)
	(c) Setting $\begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$	M1
	$\lambda = -1$	A1 (2)
	(d) Forming equations in x, y and z: $\begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 8\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	M1
	$-5x + 2y + 4z = 0$, $2x + 2z = 8y$, $4x + 2y - 5z = 0$	A1
	Establishing ratio $x : y : z$: [$x = 2y$, $x = z$]	M1
	Eigenvector $(k)\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$	A1 (4)
		[14]