Edexcel GCE

Mathematics

Pure Mathematics P1 6671

Summer 2005

advancing learning, changing lives

FINAL Mark Scheme

Edexcel GCE

Mathematics

General Instructions

1. The total number of marks for the paper is 75.

2. Method (M) marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.

3. Accuracy (A) marks can only be awarded if the relevant method (M) marks have been earned.

4. (B) marks are independent of method marks.

5. Method marks should not be subdivided.

6. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. Indicate this action by 'MR' in the body of the script (but see also note 10).

7. If a candidate makes more than one attempt at any question:

   (a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.

   (b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

8. Marks for each question, or part of a question, must appear in the right-hand margin and, in addition, total marks for each question, even where zero, must be ringed and appear in the right-hand margin and on the grid on the front of the answer book. It is important that a check is made to ensure that the totals in the right-hand margin of the ringed marks and of the unringed marks are equal. The total mark for the paper must be put on the top right-hand corner of the front cover of the answer book.

9. For methods of solution not in the mark scheme, allocate the available M and A marks in as closely equivalent a way as possible, and indicate this by the letters 'OS' (outside scheme) put alongside in the body of the script.

10. All A marks are 'correct answer only' (c.a.o.) unless shown, for example, as A1 f.t. to indicate that previous wrong working is to be followed through. In the body of the script the symbol √ should be used for correct f.t. and ✗ for incorrect f.t. After a misread, however, the subsequent A marks affected are treated as A f.t., but manifestly absurd answers should never be awarded A marks.

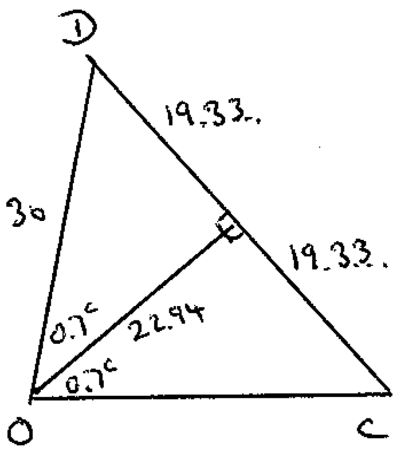
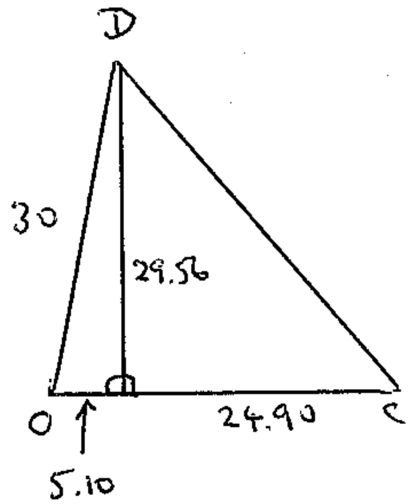
11. Ignore wrong working or incorrect statements following a correct answer.

June 2005
6671 Pure P1
Mark Scheme

Question Number	Scheme	Marks
1	(a) $\frac{dy}{dx} = 6 + 8x^{-3}$	M1 A1
		(2)
	(b) $\int y\,dx = \frac{6x^2}{2} + 4x^{-1} + C$	M1 A1A1
		(3)
		[5]
	(a) M1 is for $x^n \to x^{n-1}$ in at least one term, 6 or x^{-3} is sufficient. A1 is fully correct answer. Ignore subsequent working. (b) M1: Correct power of x in at least one term (C sufficient) First A1: $\frac{6x^2}{2} + C$ Second A1: $+4x^{-1}$	

edexcel

2	(a) $a = -4$ $x^2 - 8x - 29 \equiv (x \pm 4)^2 - 16\ \ (-29)$, $\quad b = -45$	B1 M1A1 (3)
	(b) $x - 4 = (\pm\sqrt{45})$, $\quad [\sqrt{45} = 3\sqrt{5}]$ $x = 4 \pm 3\sqrt{5}$ cao $\quad c = 4,\ d = 3$ Or: $x = \dfrac{8 \pm \sqrt{64 + 116}}{2}$, $\quad \sqrt{180} = 6\sqrt{5}$ $\quad$ M1 A1	M1 A1 A1 (3) **[6]**
	(a) B1 is for $(x-4)^2$ *or* $a = -4$ M1 requires $(x \pm p)^2 - p^2\ (\pm 29)$, $\ p \neq 0$, A1 is for $b = -45$ Answer only: full marks. Note: Can score B0M1A1 [e.g. $(x+4)^2 - 45$] Comparing coefficients: M1 is for comparing x coefficients and constant term (b) M1 is for full method leading to $x + "a" = \sqrt{.....}$ *or* $x = ...$ (formula) First A1: $c = 4$ Second A1: $d = 3$ (or -3) one of Notes: (i) If $\pm$ not seen anywhere withhold final A1, so answer only $c = 4$, $d = 3$ (or -3) (one of) is M1A1A0 (so is $4 + 3\sqrt{5}$) (ii) $x + "a" = \sqrt{-ve\ number}$ can score M1A1 for $x = 4 + ...$ (iii) using correct " quadratic formula" with $a = 1,\ b = -8,\ c = -29$ ending with $x = \dfrac{8 \pm 6\sqrt{5}}{2}$ score as M1A1A0.	

Question Number	Scheme	Marks
3	(a) $r\theta = 45\theta = 63$, $\theta = 1.4$ (*)	M1A1 (2)
	(b) Area of sector $OAB = \frac{1}{2}r^2\theta = \frac{1}{2}45^2 \times 1.4$ $(= 1417.5)$	M1A1
	Area of triangle $OCD = \frac{1}{2}30^2 \times \sin 1.4$ $(= 443.45...)$	M1A1
	Shaded area = 1417.5 – 443.45…= 974 m^2 cao	A1 (5) **[7]**
	(a) M1 is for applying correct formula or quoting and attempting to use correct formula (b) For each area M1 is for attempting to use correct formula or complete method in case of Δ * A1 is for a numerically correct statement (answer is not required – just there as check) Final A1 is for 974 only. e.g. splitting triangle into two triangles: For guidance  	

4	(a) $y - (-4) = \frac{1}{3}(x-9)$	M1A1
	$x - 3y - 21 = 0$ or $3y - x + 21 = 0$	A1 (3)
	(condone 3 terms equation: e.g. $x = 3y + 21$)	
	(b) Equation of l_2: $y = -2x$	B1
	Solve l_1 *and* l_2 simultaneously to find P:	M1
	$x = 3, \quad y = -6$	A1A1√ (4)
	(c) *C*: $(0, -7)$ or *OC* = 7 (may be on diagram)	B1√
	Area of triangle *OCP* = $\frac{1}{2} \times OC \times x_p = 10\frac{1}{2}$ (must be exact)	M1A1 (3) **[10]**
	(a) M1 for finding the equation of a straight line: If using $y - y_1 = m(x - x_1)$ or equivalent, m must be ⅓. If using $y = mx + c$, m must be ⅓ **and *c* found.** First A1: unsimplified form [$y - 9 = ⅓(x + 4)$ M0, $y - 4 = ⅓ (x -9)$ M1A0, $y - 4 = ⅓ (x + 9)$ M0] (b) M1: solving two linear equations to form linear equation in one variable A1 for first coordinate if correct A1 f.t. : For second coordinate correct for candidate after substituting in $y = -2x$ **Watch** (-3, 6) [which usually scores M1A0A1√] (c) B1 f.t. : Correct y value when $x = 0$ in candidate's equation in (a) M1: For ½ x candidate's *OC* x |candidate's x co-ord in (b)| SC: If x found when $y = 0$, allow M1 for finding area for their configuration.	

5	(a) $\arctan\frac{3}{2} = 56.3°\ (= \alpha)$ seen anywhere	B1
	$\alpha - 20°$, $(\alpha - 20°) \div 3$	M1M1
	$\alpha + 180°\ (= 236.3°)$, $\alpha - 180°\ (= -123.7°)$ (One of these)	M1
	$x = -47.9°,\ 12.1°,\ 72.1°$	A1A1 (6)
	(b) $2\sin^2 x + (1 - \sin^2 x) = \frac{10}{9}$ or $2(1 - \cos^2 x) + \cos^2 x = \frac{10}{9}$	M1
	$\sin^2 x = \frac{1}{9}$ or $\cos^2 x = \frac{8}{9}$ or $\tan^2 x = \frac{1}{8}$ or $\sec^2 x = \frac{9}{8}$ or $\cos 2x = \frac{7}{9}$	A1
	$x = 19.5°$, $-19.5°$	A1A1√ (4) **[10]**
	(a) First M1: Subtracting (allow adding) 20º from α Second M1: Dividing that result by 3 (order vital !) [**So 12.1° gains B1M1M1**] Third M1: Giving a third quadrant result First A1 is for 2 correct solutions, Second A1 for third correct solution. B1: Allow 0.983 (rads) or 62.6 (grad), and possible Ms but A0A0] (b) M1 for use of $\sin^2 x + \cos^2 x = 1$ or $\sin^2 x$ *and* $\cos^2 x$ in terms of $\cos 2x$ For use of tan $(A + B)$ see separate sheet Note : Max. deduction of 1 for not correcting to 1 dec. place. Record as 0 first time occurs but then treat as f.t. Answers outside given interval, ignore Extra answers in range, max. deduction of 1 in each part [Final mark] (i.e. 4 or more answers within interval in (a), –1 from any gained A marks; 3 or more answers within interval in (b), –1 from any gained A marks	

6	(a) $S = a + (a + d) + (a + 2d) + \ldots + [a + (n-1)d]$	B1
	$S = [a + (n-1)d] + [a + (n-2)d] + \ldots + a$ or equiv.	M1
	Add: $2S = n[2a + (n-1)d] \Rightarrow S = \frac{1}{2}n[2a + (n-1)d]$ cso (*)	M1 A1 (4)
	(b) 3, 8, 13	B1 (1)
	(c) $a = 3$ $d = 5$	B1√
	Sum $= \frac{1}{2}n[(2 \times 3) + 5(n-1)] = \frac{1}{2}n(5n+1)$ (*)	M1 A1 (3)
	(d) Finding $\sum_{1}^{200}$ e.g. $\sum_{r=1}^{200}(5r-2) = \frac{1}{2} \times 200 \times 1001$ (= 100100)	M1
	Sum of first 4 terms: $\sum_{r=1}^{4}(5r-2) = \frac{1}{2} \times 4 \times 21$ or 42 stated	B1
	$\sum_{r=5}^{200}(5r-2) = \mathrm{S}(200) - \mathrm{S}(4) = 100100 - 42 = 100058$	M1 A1 (4) **[12]**
	ALT: Working with 23, 28, 33, …………	
	$a = 23$ B1; Finding "n" and d M1	
	Applying $S = \frac{1}{2}n[2a + (n-1)d]$ with candidate's 23, n= 195 or 196 , $d = 5$ M1	
	(a) B1: requires min of 3 terms, including the last. (b) First M1 generous; second M1 hard. Note: Result is given so check working carefully. (c) For B1 f.t. 3 terms must be in AP, But allow M1 for candidate's "a" and "d" in given result in (a) (d) First M1 for substitution of 200 in result from (c) S.C. Allow second M1 for S(200) – S(5)	

7	(a) $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^{\frac{1}{2}} - 6$	M1 A1
	$3x^{\frac{1}{2}} - 6 = 0$, $x^{\frac{1}{2}} = 2$ $x = 4$ (*)	M1 A1 (4)
	(b) $\int\left(2x^{\frac{3}{2}} - 6x + 10\right)\mathrm{d}x = \left[\frac{4x^{\frac{5}{2}}}{5} - 3x^2 + 10x\right]$	M1 A1 A1
	$\left[\frac{4x^{\frac{5}{2}}}{5} - 3x^2 + 10x\right]_1^4 = \left(\frac{4\times 4^{\frac{5}{2}}}{5} - (3\times 16) + 40\right) - \left(\frac{4}{5} - 3 + 10\right)$	M1 A1√
	(= 17.6 – 7.8 = 9.8)	
	Finding area of trapezium = $\frac{1}{2}(6+2)\times 3$ $(=12)$ [A = (1, 6), B = (4, 2)]	M1 A1
	Or by integration: $\left[\frac{22x - 2x^2}{3}\right]_1^4$	
	Area of R =12 – 9.8 = 2.2	A1 (8) **[12]**
	(a) First M1 for decrease of 1 in power of x of at least one term (disappearance of “10” sufficient) Second M1 for putting $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and finding $x = \ldots$. (b) First M1: Power of at least one term increased by 1 First A1: For $\frac{4x^{5/2}}{5}$ Second A1: For $-3x^2 + 10x$ Second M1 for limits requires $\lvert[\ ]^4 - [\ ]_1\rvert$ (allow candidate’s “4”) and some processing of “integral”, $[y]_1^4$ is M0 A1√ requires **1** and **4** substituted in candidate’s **3-termed integrand** (unsimplified) Area of trapezium: M1 attempt at ½ $(y_A + y_B)(x_B - 1)$ or $\int\frac{22-4x}{3}\mathrm{d}x$ A1 correct unsimplified See separate sheet for $\int$"*line* – *curve*"	

8	(a) $\mathrm{f}(3) = 27 - 117 + 165 - 75$ (some working needed)	M1
	$= 0$, so $(x - 3)$ is a factor of $\mathrm{f}(x)$	A1 (2)
	(b) $(x-3)(x^2 - 10x + 25)$	M1 A1
	$(x-3)(x-5)(x-5)$	A1 (3)
	(c) 3 and 5	B1 (1)
	(d) $\mathrm{f}'(x) = 3x^2 - 26x + 55$	M1 A1
	$\mathrm{f}'(3) = 27 - 78 + 55 = 4$	A1 (3)
	(e) "$3x^2 - 26x + 55$" = "4"	M1
	$3x^2 - 26x + 51 = 0 \Rightarrow (3x-17)(x-3) = 0$ $x = \ldots$ dep	M1 A1√
	x-coordinate of S is $\frac{17}{3}$ $\left(\frac{34}{6}\ or\ 5\frac{2}{3}\ or\ 5.\dot{6}\ or\ 5.67\right)$	A1 (4) **[13]**
	(a) M1 is for substituting $x = 3$ in $\mathrm{f}(x)$ A1 requires $\mathrm{f}(3) = 0$ and statement (b) M1 for quadratic factor $(x^2 + ax \pm 25)$ SC: M1 for one other linear factor found (d) M1 for differentiation: at least one term has power of x reduced by 1 (e) First M1 : equating their gradient function to their answer to (d) Second M1: Solving by factors requires $(ax^2 + bx + c) = (mx + p)(nx + q)$ *where* $\lvert pq \rvert = \lvert c \rvert$ *and* $\lvert mn \rvert = \lvert a \rvert$, *leading to* $x = \ldots$ Solving by quadratic formula requires attempt to use correct formula with candidate's values of *a*, *b* and *c* used. A1 f.t. only for correct (real) answers to their quadratic Final A1 for a correct exact form.	

	EXTRAS	
5(a)	Using expansion of $\tan(3x + 20^\circ) = \frac{3}{2}$	
	Getting as far as $\tan 3x$ = number (0.7348..)	First M1
	$\tan 3x = 36.3^o, \quad 216.3^o, \ -143.7^o$ 36.3^o	B1
	$x = 12.1\ ^o, \ 72.1\ ^o, \ -\ 47.9\ ^o$ Third quad result	Third M1
	Divide by 3	Second M1
	Answers as scheme	A1A1
6(c)	$5\sum r \ - \ \sum 2$	B1
	$= \ 5\,\frac{n(n+1)}{2} - 2n$	M1
	$= \ \frac{5n^2 + n}{2} \ = \frac{n(5n+1)}{2}$ * (cso)	A1
7(b)	Attempting integral |(equation of line – equation of curve)!	Third M1
	$= \ \int \lvert\,(-\frac{8}{3} + \frac{14}{3}x - 2x^{\frac{3}{2}})\,\rvert \, dx$	Fourth A1
	Performing integration:	First M1
	$\left[(-\frac{8}{3}x + \frac{7}{3}x^2) - (\frac{4}{5}x^{\frac{5}{2}})\right]$ $\left[(\frac{4}{5}x^{\frac{5}{2}})\right]$	First A1
	$\left[\lvert\,(-\frac{8}{3}x + \frac{7}{3}x^2)\rvert\right]$ allow as follow through in this case.	Second A1
	Limits M1A1$\sqrt{\ }$	Second M1 Third A1 Fifth A1
	Answer A1	

GENERAL PRINCIPLES FOR P1 MARKING

Method mark for solving 3 term quadratic:

1. Factorisation

   $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

   $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

   Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

   Solving $x^2 + bx + c = 0$: $(x \pm p)^2 \pm q \pm c$, $p \neq 0$, $q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

   Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

   Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but will be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may gain no credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Misreads

(See the next sheet for a simple example).
A misread must be consistent for the whole question to be interpreted as such.
These are not common. In clear cases, please deduct the first 2 A (or B) marks which would have been lost by following the scheme. (Note that 2 marks is the maximum misread penalty, but that misreads which alter the nature or difficulty of the question cannot be treated so generously and it will usually be necessary here to follow the scheme as written).
Sometimes following the scheme as written is more generous to the candidate than applying the misread rule, so in this case use the scheme as written.