Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Thursday 30 June 2005 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Lilac or Green)	Nil
Answer Book (AB16)	
Graph paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and parts of questions are shown in round brackets: e.g. **(2)**.
This paper has 7 questions. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. A small smooth ball of mass $\frac{1}{2}$ kg is falling vertically. The ball strikes a smooth plane which is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. Immediately before striking the plane the ball has speed 10 m s^{-1}. The coefficient of restitution between ball and plane is $\frac{1}{2}$. Find

(*a*) the speed, to 3 significant figures, of the ball immediately after the impact,

(5)

(*b*) the magnitude of the impulse received by the ball as it strikes the plane.

(2)

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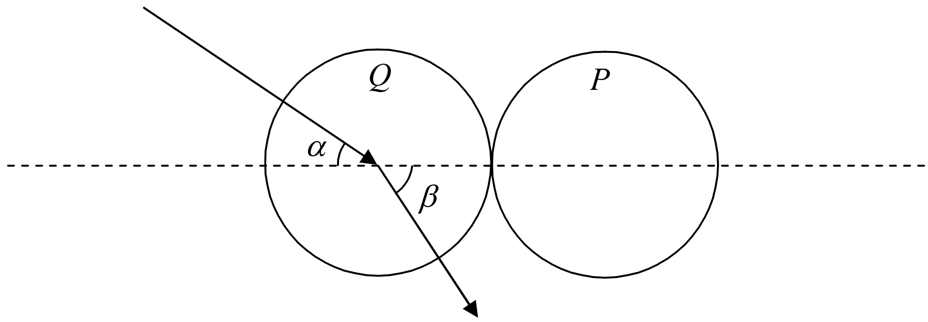
2. A cyclist P is cycling due north at a constant speed of 20 km h^{-1}. At 12 noon another cyclist Q is due west of P. The speed of Q is constant at 10 km h^{-1}. Find the course which Q should set in order to pass as close to P as possible, giving your answer as a bearing.

(5)

3.

Figure 1



A smooth sphere P lies at rest on a smooth horizontal plane. A second identical sphere Q, moving on the plane, collides with the sphere P. Immediately before the collision the direction of motion of Q makes an angle α with the line joining the centres of the spheres. Immediately after the collision the direction of motion of Q makes an angle β with the line joining the centres of spheres, as shown in Figure 1. The coefficient of restitution between the spheres is e.

Show that $(1 - e) \tan \beta = 2 \tan \alpha$.

(11)

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4. A lorry of mass M is moving along a straight horizontal road. The engine produces a constant driving force of magnitude F. The total resistance to motion is modelled as having magnitude kv^2, where k is a constant, and v is the speed of the lorry.

Given the lorry moves with constant speed V,

(a) show that $V = \sqrt{\dfrac{F}{k}}$.

(2)

Given instead that the lorry starts from rest,

(b) show that the distance travelled by the lorry in attaining a speed of $\frac{1}{2}V$ is

$$\frac{M}{2k}\ln\left(\frac{4}{3}\right).$$

(9)

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5. A non-uniform rod BC has mass m and length $3l$. The centre of mass of the rod is at distance l from B. The rod can turn freely about a fixed smooth horizontal axis through B. One end of a light elastic string, of natural length l and modulus of elasticity $\dfrac{mg}{6}$, is attached to C. The other end of the string is attached to a point P which is at a height $3l$ vertically above B.

(a) Show that, while the string is stretched, the potential energy of the system is

$$mgl(\cos^2\theta - \cos\theta) + \text{constant},$$

where θ is the angle between the string and the downward vertical and $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$.

(6)

(b) Find the values of θ for which the system is in equilibrium with the string stretched.

(6)

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6. A ship A has maximum speed 30 km h^{-1}. At time $t = 0$, A is 70 km due west of B which is moving at a constant speed of 36 km h^{-1} on a bearing of 300°. Ship A moves on a straight course at a constant speed and intercepts B. The course of A makes an angle θ with due north.

(a) Show that $-\arctan\frac{4}{3} \leq \theta \leq \arctan\frac{4}{3}$.

(7)

(b) Find the least time for A to intercept B.

(5)

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7. A light elastic string, of natural length a and modulus of elasticity $5ma\omega^2$, lies unstretched along a straight line on a smooth horizontal plane. A particle of mass m is attached to one end of the spring. At time $t = 0$, the other end of the spring starts to move with constant speed U along the line of the spring and away from the particle. As the particle moves along the plane it is subject to a resistance of magnitude $2m\omega v$, where v is its speed. At time t, the extension of the spring is x and the displacement of the particle from its initial position is y. Show that

(*a*) $x + y = Ut$,

(2)

(*b*) $\dfrac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 2\omega\dfrac{\mathrm{d}x}{\mathrm{d}t} + 5\omega^2 x = 2\omega U$.

(7)

(*c*) Find x in terms of ω, U and t.

(8)

TOTAL FOR PAPER: 75 MARKS

END