Paper Reference(s)

6675/01

Edexcel GCE

Pure Mathematics P5
Further Mathematics FP2
Advanced/Advanced Subsidiary

Monday 20 June 2005 – Morning
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Lilac)	Nil
Graph Paper (ASG2)	
Answer Book (AB16)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P5), the paper reference (6675), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has 8 questions.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N21141A

1. (*a*) Find $\int \frac{1+x}{\sqrt{(1-4x^2)}}\,dx$.

(5)

(*b*) Find, to 3 decimal places, the value of

$$\int_0^{0.3} \frac{1+x}{\sqrt{(1-4x^2)}}\,dx.$$

(2)

(Total 7 marks)

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2. (*a*) Show that, for $x = \ln k$, where k is a positive constant,

$$\cosh 2x = \frac{k^4+1}{2k^2}.$$

(3)

Given that $f(x) = px - \tanh 2x$, where p is a constant,

(*b*) find the value of p for which $f(x)$ has a stationary value at $x = \ln 2$, giving your answer as an exact fraction.

(4)

(Total 7 marks)

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3. **Figure 1**

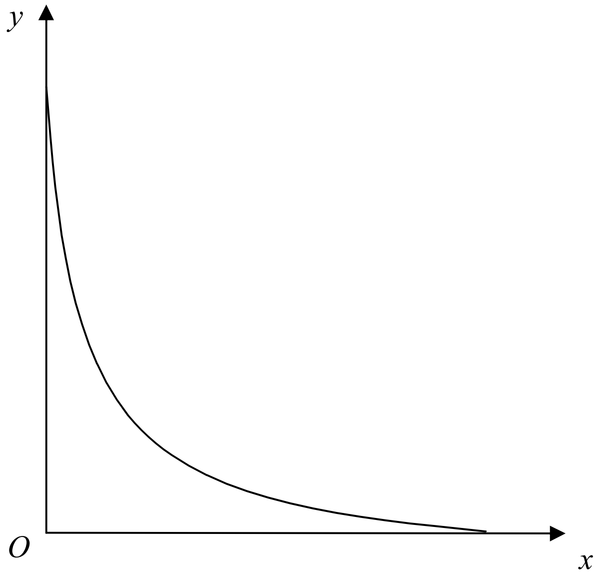

Figure 1 shows a sketch of the curve with parametric equations

$$x = a\cos^3 t, \qquad y = a\sin^3 t, \qquad 0 \le t \le \frac{\pi}{2},$$

where a is a positive constant.

The curve is rotated through 2π radians about the x-axis. Find the exact value of the area of the curved surface generated.

(Total 7 marks)

4. $$I_n = \int x^n e^{2x}\, dx, \quad n \geq 0.$$

(*a*) Prove that, for $n \geq 1$,

$$I_n = \frac{1}{2}(x^n e^{2x} - nI_{n-1}).$$

(3)

(*b*) Find, in terms of e, the exact value of

$$\int_0^1 x^2 e^{2x} \quad dx.$$

(5)

(Total 8 marks)

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5. The point $P(ap^2, 2ap)$ lies on the parabola M with equation $y^2 = 4ax$, where a is a positive constant.

(*a*) Show that an equation of the tangent to M at P is

$$py = x + ap^2.$$

(3)

The point $Q(16ap^2, 8ap)$ also lies on M.

(*b*) Write down an equation of the tangent to M at Q.

(2)

The tangent at P and the tangent at Q intersect at the point V.

(*c*) Show that, as p varies, the locus of V is a parabola N with equation

$$4y^2 = 25ax.$$

(4)

(*d*) Find the coordinates of the focus of N, and find an equation of the directrix of N.

(2)

(*e*) Sketch M and N on the same diagram, labelling each of them.

(2)

(Total 13 marks)

6.

Figure 2

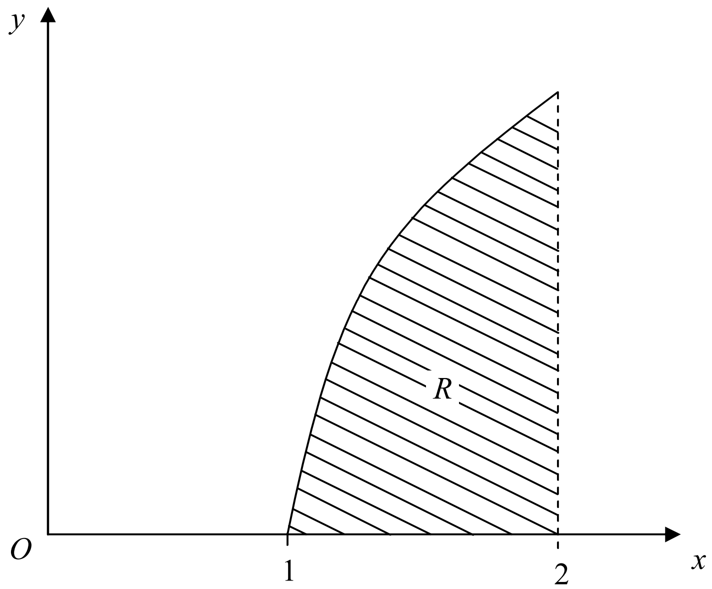

Figure 2 shows a sketch of the curve with equation

$$y = x \text{ arcosh } x, \qquad 1 \le x \le 2.$$

The region R, as shown shaded in Figure 2, is bounded by the curve, the x-axis and the line $x = 2$.

Show that the area of R is

$$\frac{7}{4} \ln (2 + \sqrt{3}) - \frac{\sqrt{3}}{2}.$$

(Total 10 marks)

7. The curve C has parametric equations

$$x = t + \sin t, \quad y = 1 - \cos t, \quad 0 \leq t < \frac{\pi}{2}.$$

The arc length s of the curve C is measured from the origin O.

Show that

(*a*) $s = 4 \sin \frac{t}{2}$,

(4)

(*b*) an intrinsic equation of C is $s = 4 \sin \psi$.

(4)

Hence, or otherwise,

(*c*) find the radius of curvature of C at the point for which $t = \frac{\pi}{3}$.

(2)

(Total 10 marks)

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8. (*a*) Show that, for $0 < x \leq 1$,

$$\ln\left(\frac{1 - \sqrt{(1 - x^2)}}{x}\right) = -\ln\left(\frac{1 + \sqrt{(1 - x^2)}}{x}\right).$$

(3)

(*b*) Using the definition of cosh x or sech x in terms of exponentials, show that, for $0 < x \leq 1$,

$$\text{arsech } x = \ln\left(\frac{1 + \sqrt{(1 - x^2)}}{x}\right).$$

(5)

(*c*) Solve the equation

$$3 \tanh^2 x - 4 \text{ sech } x + 1 = 0,$$

giving exact answers in terms of natural logarithms.

(5)

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END