Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Subsidiary

Monday 20 June 2005 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. (*a*) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \tan^2\theta \equiv \sec^2\theta$.

(2)

(*b*) Solve, for $0 \leq \theta < 360°$, the equation

$$2\tan^2\theta + \sec\theta = 1,$$

giving your answers to 1 decimal place.

(6)

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2. (*a*) Differentiate with respect to x

(i) $3\sin^2 x + \sec 2x$,

(3)

(ii) $\{x + \ln(2x)\}^3$.

(3)

Given that $y = \dfrac{5x^2 - 10x + 9}{(x-1)^2}$, $x \neq 1$,

(*b*) show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{8}{(x-1)^3}$.

(6)

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3. The function f is defined by

$$\mathrm{f}\colon x \mapsto \frac{5x+1}{x^2+x-2} - \frac{3}{x+2}, \quad x > 1.$$

(*a*) Show that $\mathrm{f}(x) = \dfrac{2}{x-1}$, $x > 1$.

(4)

(*b*) Find $\mathrm{f}^{-1}(x)$.

(3)

The function g is defined by

$$\mathrm{g}\colon x \mapsto x^2 + 5, \quad x \in \mathbb{R}.$$

(*b*) Solve $\mathrm{fg}(x) = \frac{1}{4}$.

(3)

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4. $$f(x) = 3e^x - \tfrac{1}{2}\ln x - 2, \quad x > 0.$$

(*a*) Differentiate to find $f'(x)$.

(3)

The curve with equation $y = f(x)$ has a turning point at P. The x-coordinate of P is α.

(*b*) Show that $\alpha = \tfrac{1}{6}e^{-\alpha}$.

(2)

The iterative formula

$$x_{n+1} = \tfrac{1}{6}e^{-x_n}, \quad x_0 = 1,$$

is used to find an approximate value for α.

(*c*) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

(2)

(*d*) By considering the change of sign of $f'(x)$ in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places.

(2)

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5. (*a*) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A.$$

(2)

(*b*) Show that

$$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv \sin\theta(4\cos\theta + 6\sin\theta - 3).$$

(4)

(*c*) Express $4\cos\theta + 6\sin\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \tfrac{1}{2}\pi$.

(4)

(*d*) Hence, for $0 \le \theta < \pi$, solve

$$2\sin 2\theta = 3(\cos 2\theta + \sin\theta - 1),$$

giving your answers in radians to 3 significant figures, where appropriate.

(5)

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6. **Figure 1**

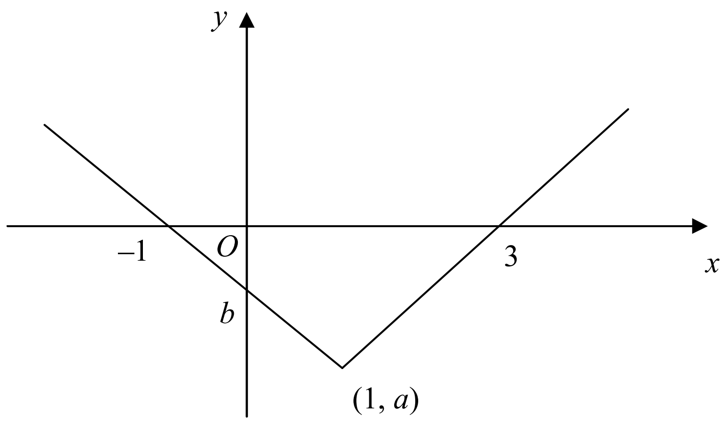

Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the x-axis at $(3, 0)$. The other line meets the x-axis at $(–1, 0)$ and the y-axis at $(0, b)$, $b < 0$.

In separate diagrams, sketch the graph with equation

(*a*) $y = f(x + 1)$,
(2)

(*b*) $y = f(|x|)$.
(3)

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $f(x) = |x - 1| - 2$, find

(*c*) the value of a and the value of b,
(2)

(*d*) the value of x for which $f(x) = 5x$.
(4)

7. A particular species of orchid is being studied. The population p at time t years after the study started is assumed to be

$$p = \frac{2800a\mathrm{e}^{0.2t}}{1 + a\mathrm{e}^{0.2t}}, \text{ where } a \text{ is a constant.}$$

Given that there were 300 orchids when the study started,

(*a*) show that $a = 0.12$,

(3)

(*b*) use the equation with $a = 0.12$ to predict the number of years before the population of orchids reaches 1850.

(4)

(*c*) Show that $p = \dfrac{336}{0.12 + \mathrm{e}^{-0.2t}}$.

(1)

(*d*) Hence show that the population cannot exceed 2800.

(2)

TOTAL FOR PAPER: 75 MARKS

END