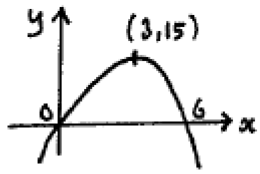
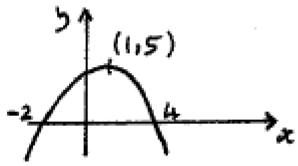
June 2005
6663 Core Mathematics C1
Mark Scheme

Question Number	Scheme	Marks
1. (a)	$\underline{2}$ Penalise $\pm$	B1 (1)
(b)	$8^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{64}}$ or $\frac{1}{(a)^2}$ or $\frac{1}{\sqrt[3]{8^2}}$ or $\frac{1}{8^{\frac{2}{3}}}$ Allow $\pm$	M1
	$= \frac{1}{4}$ or 0.25	A1 (2)
		(3)
(b)	M1 for understanding that "-" power means reciprocal $8^{\frac{2}{3}} = 4$ is M0A0 and $-\frac{1}{4}$ is M1A0	
2. (a)	$\frac{dy}{dx} = 6 + 8x^{-3}$ $x^n \to x^{n-1}$	M1
	both ($6x^0$ is OK)	A1 (2)
(b)	$\int (6x - 4x^{-2})dx = \frac{6x^2}{2} + 4x^{-1} + c$	M1 A1 A1 (3)
		(5)
(b)	In (a) and (b) M1 is for a correct power of x in at least one term. This could be 6 in (a) or $+c$ in (b) 1st A1 for one correct term in x: $\frac{6x^2}{2}$ <u>or</u> $+4x^{-1}$ (or better simplified versions) 2nd A1 for all 3 terms as printed or better in one line. N.B. M1A0A1 is not possible. SC. For integrating their answer to part (a) just allow the M1 if $+c$ is present	

Question Number	Scheme	Marks
3. (a)	$x^2 - 8x - 29 \equiv (x-4)^2 - 45$ $\quad$ $(x \pm 4)^2$ $(x-4)^2 - 16 + (-29)$ $(x \pm 4)^2 - 45$	M1 A1 A1 (3)
ALT	Compare coefficients $\quad -8 = 2a \quad$ equation for a $a = -4$ <u>AND</u> $a^2 + b = -29$ $b = -45$	M1 A1 A1 (3)
(b)	$(x-4)^2 = 45$ $\quad$ (follow through their a and b from (a)) $\Rightarrow x - 4 = \pm\sqrt{45}$ $\quad$ $c = 4$ $x = 4 \pm 3\sqrt{5}$ $\quad$ $d = 3$ ($\pm$ OK)	M1 A1 A1 (3) **(6)**
(a) (b)	M1 for $(x \pm 4)^2$ or an equation for a (allow sign error ± 4 or ± 8 on ALT) 1stA1 for $(x-4)^2 - 16(-29)$ can ignore -29 <u>or</u> for stating $a = -4$ and an equation for b 2nd A1 for $b = -45$ Note M1A0 A1 is possible for $(x+4)^2 - 45$ **N.B. On EPEN these marks are called B1M1A1 but apply them as M1A1A1** M1 for a full method leading to $x - 4 = ...$ or $x = ...$ (condone $x - 4 = \sqrt{-n}$) N.B. $(x-4)^2 - 45 = 0$ leading to $(x-4) \pm \sqrt{45} = 0$ is M0A0A0 A1 for c and A1 for d N.B. M1 and A1 for c do not need $\pm$ (so this is a special case for the formula method) but $\pm$ must be present for the d mark) <u>Note</u> Use of formula that ends with $\frac{8 \pm 6\sqrt{5}}{2}$ scores M1 A1 A0 (but must be $\sqrt{5}$) i.e. only penalise non-integers by one mark.	

Question Number	Scheme	Marks
4. (a)	Shape Points 	B1 B1 (2)
(b)	 -2 and 4 max	M1 A1 A1 (3) **(5)**
(a) (b)	Marks for shape: graphs must have curved sides and round top. Don't penalise twice. (If both graphs are really straight lines then penalise B0 in part (a) only) 1st B1 for $\cap$ shape through (0, 0) and $((k,0)$ where $k>0)$ 2nd B1 for max at (3, 15) and 6 labelled or (6, 0) seen Condone (15,3) if 3 and 15 are correct on axes. Similarly (5,1) in (b) M1 for $\cap$ shape NOT through (0, 0) but must cut x-axis twice. 1st A1 for -2 and 4 labelled or (-2, 0) and (4, 0) seen 2nd A1 for max at (1, 5). Must be clearly in 1st quadrant	
5.	$x = 1 + 2y$ and sub $\rightarrow (1+2y)^2 + y^2 = 29$ $\Rightarrow 5y^2 + 4y - 28(=0)$ i.e. $(5y+14)(y-2) = 0$ $(y=)2$ or $-\frac{14}{5}$ (o.e.) (both) $y = 2 \Rightarrow x = 1 + 4 = 5$; $y = -\frac{14}{5} \Rightarrow x = -\frac{23}{5}$ (o.e)	M1 A1 M1 A1 M1A1 f.t. **(6)**
	1st M1 Attempt to sub leading to equation in 1 variable Condone sign error such as $1-2y$, x = -(1+2y) penalise 1st A1 only 1st A1 Correct 3TQ (condone = 0 missing) 2nd M1 Attempt to solve 3TQ leading to 2 values for y. 2nd A1 Condone mislabelling x = for y = … but then M0A0 in part (c). 3rd M1 Attempt to find at least one x value (must use a correct equation) 3rd A1 f.t. f.t. only in $x = 1 + 2y$ (3sf if not exact) Both values. N.B False squaring. (e.g. $x^2 + 4y^2 = 1$) can only score the last 2 marks.	

Question Number	Scheme	Marks
6. (a)	$6x+3 > 5-2x \quad \Rightarrow 8x > 2$	M1
	$x > \frac{1}{4}$ or 0.25 or $\frac{2}{8}$	A1
		(2)
(b)	$(2x-1)(x-3) \quad (>0)$	M1
	Critical values $x = \frac{1}{2},\ 3$ (both)	A1
		
	Choosing "outside" region	M1
	$x > 3$ or $x < \frac{1}{2}$	A1 f.t.
		(4)
(c)	$x>3$ or $\frac{1}{4} < x < \frac{1}{2}$ $[(3,\infty)$ or $(\frac{1}{4},\frac{1}{2})$ is OK]	B1f.t. B1f.t.
		(2)
		(8)
(a)	M1 Multiply out and collect terms (allow one slip and allow use of = here)	
(b)	1st M1 Attempting to factorise 3TQ $\rightarrow x = ...$	
	2nd M1 Choosing the outside region	
	2nd A1 f.t. f.t. their critical values N.B.($x>3$, $x > \frac{1}{2}$ is M0A0)	
(c)	**f.t. their answers to (a) and (b)**	
	1st B1 a correct f.t. leading to an <u>infinite</u> region 2nd B1 a correct f.t. leading to a <u>finite</u> region	
	Penalise $\le$ or $\ge$ once only at first offence. For $p < x < q$ where $p > q$ penalise the final A1 in (b) .	
	e.g. (a) (b) (c) Mark	
	$x > \frac{1}{4}$ $\frac{1}{2} < x < 3$ $\frac{1}{2} < x < 3$ B0 B1	
	$x > \frac{1}{4}$ $x > 3,\ x > \frac{1}{2}$ $x > 3$ B1 B0	

Question Number	Scheme	Marks
7. (a)	$(3-\sqrt{x})^2 = 9-6\sqrt{x}+x$	M1
	$\div by\sqrt{x} \quad \rightarrow 9x^{-\frac{1}{2}}-6+x^{\frac{1}{2}}$	A1 c.s.o.
		(2)
(b)	$\int(9x^{-\frac{1}{2}}-6+x^{\frac{1}{2}})dx = \frac{9x^{\frac{1}{2}}}{\frac{1}{2}}-6x+\frac{x^{\frac{3}{2}}}{\frac{3}{2}}(+c)$	M1 A2/1/0
	use $y=\frac{2}{3}$ and $x=1$: $\quad \frac{2}{3}=18\ -6+\frac{2}{3}+c$	M1
	$c = -12$	A1 c.s..o.
	So $\quad y=18x^{\frac{1}{2}}-6x+\frac{2}{3}x^{\frac{3}{2}}$ -12	A1f.t.
		(6)
		(8)
(a)	M1 Attempt to multiply out $(3-\sqrt{x})^2$. Must have 3 or 4 terms, allow one sign error A1 cso Fully correct solution to printed answer. Penalise invisible brackets or wrong working	
(b)	1st M1 Some correct integration: $x^n \rightarrow x^{n+1}$ A1 At least 2 correct unsimplified terms Ignore + c A2 All 3 terms correct (unsimplified) 2nd M1 Use of $y=\frac{2}{3}$ and $x=1$ to find c. No + c is M0. A1c.s.o. for -12. (o.e.) Award this mark if " $c=-12$ " stated i.e. not as part of an expression for y A1f.t. for 3 simplified x terms with $y=\ldots$ and a numerical value for c. Follow through their value of c but it must be a number.	
Question	Scheme	Marks

Number		
8. (a)	$y-(-4)=\frac{1}{3}(x-9)$ or $\frac{y-(-4)}{x-9}=\frac{1}{3}$ $3y-x+21=0$ (o.e.) (condone 3 terms with integer coefficients e.g. $3y+21=x$)	M1 A1 A1 (3)
(b)	Equation of l_2 is: $y=-2x$ (o.e.) Solving l_1 and l_2 : $-6x-x+21=0$ p is point where $x_p=3$, $y_p=-6$ x_p or y_p y_p or x_p	B1 M1 A1 A1f.t. $(-2x)$ (4)
(c) ALT	(l_1 is $y=\frac{1}{3}x-7$) C is (0, -7) or OC = 7 Area of $\Delta OCP=\frac{1}{2}OC\times x_p$, $=\frac{1}{2}\times7\times3=10.5$ or $\frac{21}{2}$ By Integration: M1 for $\pm\int_0^{x_P}(l_1-l_2)\,dx$, B1 ft for correct integration (follow through their l_1), then A1cao.	B1f.t. M1 A1c.a.o. (3) **(10)**
(a)	M1 for full method to find equation of l_1 1stA1 any unsimplified form	
(b)	M1 Attempt to solve two linear equations leading to linear equation in one variable 2nd A1 f.t. only f.t. their x_p or y_p in $y=-2x$ N.B. A fully correct solution by drawing, or correct answer with no working can score all the marks in part (b), but a partially correct solution by drawing only scores the first B1.	
(c)	B1f.t. Either a correct OC or f.t. from their l_1 M1 for correct attempt in letters or symbols for ΔOCP A1 c.a.o. $-\frac{1}{2}\times7\times3$ scores M1 A0	
MR	(x-axis for y-axis) Get $C=(21, 0)$ Area of $\Delta OCP=\frac{1}{2}OC\times y_p$ $=\frac{1}{2}\times21\times6=63$ (B0M1A0)	

Question Number	Scheme	Marks
9 (a)	$(S=)a+(a+d)+\ldots \quad \ldots+[a+(n-1)d]$ $(S=)[a+(n-1)d]+\ldots \quad \ldots+a$ $2S=[2a+(n-1)d]+\ldots \quad \ldots+[2a+(n-1)d]$ } either $2S=n[2a+(n-1)d]$ $S=\frac{n}{2}[2a+(n-1)d]$	B1 M1 dM1 A1 c.s.o (4)
(b)	$(a=149,\ d=-2)$ $u_{21}=149+20(-2)\ =£109$	M1 A1 (2)
(c)	$S_n=\frac{n}{2}[2\times149+(n-1)(-2)] \qquad (=n(150-n))$ $S_n=5000 \Rightarrow n^2-150n+5000=0 \quad (*)$	M1 A1 A1 c.s.o (3)
(d)	$(n-100)(n-50)=0$ $n=50$ or 100	M1 A2/1/0 (3)
(e)	$u_{100}<0 \qquad \therefore n=100$ not sensible	B1 f.t. (1) **(13)**
(a)	B1 requires at least 3 terms, must include first and last terms, an adjacent term and dots! There must be + signs for the B1 (or at least implied see snippet 9D) 1st M1 for reversing series. Must be arithmetic with *a*, *n* and *d* or *l*. (+ signs not essential here) 2nd dM1 for adding, must have 2*S* and be a genuine attempt. Either line is sufficient. Dependent on 1st M1 (NB Allow first 3 marks for use of *l* for last term but as given for final mark)	
(b)	M1 for using a = 149 and $d=\pm 2$ in $a+(n-1)d$ formula.	
(c)	M1 for using their a,d in S_n A1 any correct expression A1cso for putting S_n=5000 and simplifying to given expression. No wrong work **NB EPEN has B1M1A1 here but apply marks as M1A1A1 as in scheme**	
(d)	M1 Attempt to solve leading to $n=\ldots$ A2/1/0 Give A1A0 for 1 correct value and A1A1 for both correct	
(e)	B1 f.t. Must mention 100 and state $u_{100}<0$ (or loan paid or equivalent) If giving f.t. then must have $n\geq 76$.	

Question Number	Scheme	Marks
10 (a)	$x = 3, \quad y = 9 - 36 + 24 + 3 = 0$ (9 – 36 + 27=0 is OK)	B1 (1)
(b)	$\frac{dy}{dx} = \frac{3}{3}x^2 - 2\times 4\times x + 8 \qquad (x^2 - 8x + 8)$	M1 A1
	When $x = 3$, $\frac{dy}{dx} = 9 - 24 + 8 \Rightarrow m = -7$	M1
	Equation of tangent: $y - 0 = -7(x - 3)$	M1
	$y = -7x + 21$	A1 c.a.o (5)
(c)	$\frac{dy}{dx} = m$ gives $x^2 - 8x + 8 = -7$	M1
	$(x^2 - 8x + 15 = 0)$	
	$(x-5)(x-3) = 0$	M1
	$x = (3)$ or 5 5	A1
	$\therefore y = \frac{1}{3}5^3 - 4\times 5^2 + 8\times 5 + 3$	M1
	$y = -15\frac{1}{3}$ or $-\frac{46}{3}$	A1 (5)
		(11)
(b)	1st M1 some correct differentiation ($x^n \to x^{n-1}$ for one term) 1st A1 correct unsimplified (all 3 terms) 2nd M1 substituting $x_P(=3)$ in their $\frac{dy}{dx}$ clear evidence 3rd M1 using their m to find tangent at p .The m must be from their $\frac{dy}{dx}$ at $x_P(=3)$ Use of $\frac{1}{7}$ here scores M0A0 but Could get all 3 Ms in Part (c).	
(c)	1st M1 forming a correct equation " their $\frac{dy}{dx}$ = gradient of their tangent" 2nd M1 for solving a quadratic based on their $\frac{dy}{dx}$ leading to $x = ...$ The quadratic could be simply $\frac{dy}{dx}$ =0. 3rd M1 for using their x value (obtained from their quadratic) in y to obtain y coordinate. Must have one of the other two M marks to score this.	
MR	For misreading (0, 3) for (3, 0) award B0 and then M1A1 as in scheme. Then allow all M marks but no A ft. (Max 7)	