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Question 1 continued

Handwriting practice lines for the answer to Question 1.

Q1

(Total 6 marks)



3. A river, running between parallel banks, is 20 m wide. The depth, y metres, of the river measured at a point x metres from one bank, is given by the formula

$$y = \frac{1}{10}x\sqrt{(20-x)}, \quad 0 \leq x \leq 20.$$

- (a) Complete the table below, giving values of y to 3 decimal places.

x	0	4	8	12	16	20
y	0		2.771			0

(2)

- (b) Use the trapezium rule with all the values in the table to estimate the cross-sectional area of the river.

(4)

Given that the cross-sectional area is constant and that the river is flowing uniformly at 2 m s^{-1} ,

- (c) estimate, in m^3 , the volume of water flowing per minute, giving your answer to 3 significant figures.

(2)



Question 4 continued

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Lined writing area for the answer to Question 4.

(Total 10 marks)

Q4

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5.

Figure 1

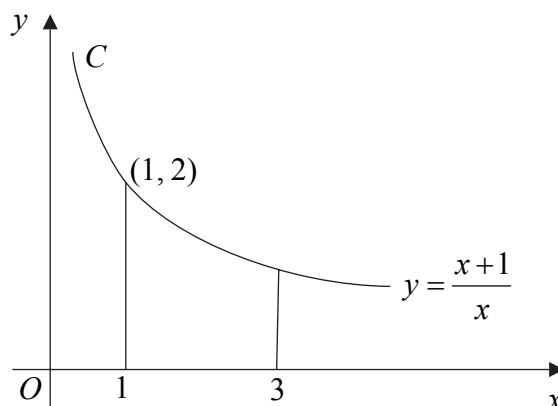


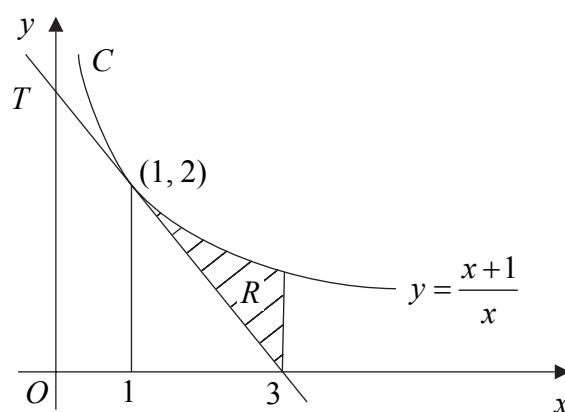
Figure 1 shows part of the curve C with equation $y = \frac{x+1}{x}$, $x > 0$.

The finite region enclosed by C , the lines $x = 1$, $x = 3$, and the x -axis is rotated through 360° about the x -axis to generate a solid S .

(a) Using integration, find the exact volume of S .

(7)

Figure 2



The tangent T to C at the point $(1, 2)$ meets the x -axis at the point $(3, 0)$. The shaded region R is bounded by C , the line $x = 3$ and T , as shown in Figure 2.

(b) Using your answer to part (a), find the exact volume generated by R when it is rotated through 360° about the x -axis.

(3)



Question 5 continued

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Lined writing area for the answer to Question 5.



6. $f(x) = 3e^x - \frac{1}{2} \ln x - 2, \quad x > 0.$

- (a) Differentiate to find $f'(x)$. **(3)**

The curve with equation $y = f(x)$ has a turning point P . The x -coordinate of P is α .

- (b) Show that $\alpha = \frac{1}{6}e^{-\alpha}$. **(2)**

The iterative formula

$$x_{n+1} = \frac{1}{6}e^{-x_n}, \quad x_0 = 1,$$

is used to find an approximate value for α .

- (c) Calculate the values of x_1, x_2, x_3 and x_4 , giving your answers to 4 decimal places. **(2)**

- (d) By considering the change of sign of $f'(x)$ in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places. **(2)**



7.

Figure 3

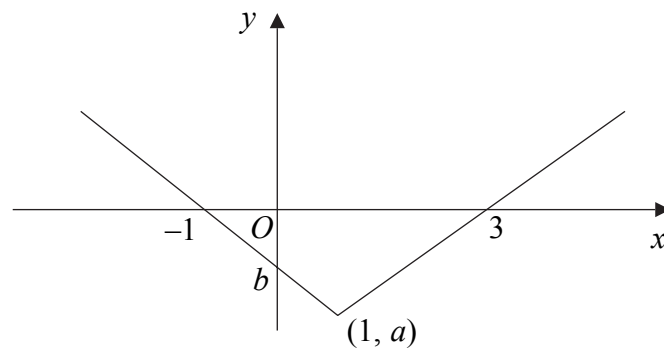


Figure 3 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the x -axis at $(3, 0)$. The other line meets the x -axis at $(-1, 0)$ and the y -axis at $(0, b)$, $b < 0$.

In separate diagrams, sketch the graph with equation

(a) $y = f(x + 1)$, (2)

(b) $y = f(|x|)$. (3)

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $f(x) = |x - 1| - 2$, find

(c) the value of a and the value of b , (2)

(d) the value of x for which $f(x) = 5x$. (4)



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