GCSE

Edexcel GCE

Core Mathematics C4 (6666)

Summer 2005

advancing learning, changing lives

Mark Scheme (Results)

Edexcel GCE
Core Mathematics C4 (6666)

Final Version

June 2005
6666 Core C4
Mark Scheme

Question Number	Scheme	Marks
1.	$(4-9x)^{\frac{1}{2}} = 2\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$	B1
	$= 2\left(1+\frac{\frac{1}{2}}{1}\left(-\frac{9x}{4}\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}\left(-\frac{9x}{4}\right)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2.3}\left(-\frac{9x}{4}\right)^3+\ldots\right)$	M1
	$= 2\left(1-\frac{9}{8}x-\frac{81}{128}x^2-\frac{729}{1024}x^3+\ldots\right)$	
	$= 2-\frac{9}{4}x, -\frac{81}{64}x^2, -\frac{729}{512}x^3+\ldots$	A1, A1, A1
		[5]
	Note The M1 is gained for $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}(\ \ldots\ )^2$ or $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2.3}(\ \ldots\ )^3$	
	Special Case	
	If the candidate reaches $= 2\left(1-\frac{9}{8}x-\frac{81}{128}x^2-\frac{729}{1024}x^3+\ldots\right)$ and goes no further	
	allow A1 A0 A0	

Question Number	Scheme	Marks
2.	$2x+\left(2x\frac{dy}{dx}+2y\right)-6y\frac{dy}{dx}=0$	M1 (A1) A1
	$\frac{dy}{dx}=0 \quad \Rightarrow \quad x+y=0$ or equivalent	M1
	Eliminating either variable and solving for at least one value of x or y.	M1
	$y^2-2y^2-3y^2+16=0$ or the same equation in x	
	$y=\pm 2$ or $x=\pm 2$	A1
	$(2,-2),(-2,2)$	A1
		[7]
	Note: $\frac{dy}{dx}=\frac{x+y}{3y-x}$	
	Alternative	
	$3y^2-2xy-\left(x^2+16\right)=0$	
	$y=\frac{2x\pm\sqrt{\left(16x^2+192\right)}}{6}$	
	$\frac{dy}{dx}=\frac{1}{3}\pm\frac{1}{3}.\frac{8x}{\sqrt{\left(16x^2+192\right)}}$	M1 A1± A1
	$\frac{dy}{dx}=0 \quad \Rightarrow \quad \frac{8x}{\sqrt{\left(16x^2+192\right)}}=\pm 1$	M1
	$64x^2=16x^2+192$	
	$x=\pm 2$	M1 A1
	$(2,-2),(-2,2)$	A1
		[7]

Question Number	Scheme	Marks
3.	(a) $\frac{5x+3}{(2x-3)(x+2)} = \frac{A}{2x-3} + \frac{B}{x+2}$	
	$5x+3 = A(x+2) + B(2x-3)$	
	Substituting $x = -2$ or $x = \frac{3}{2}$ and obtaining *A* or *B*; or equating coefficients and solving a pair of simultaneous equations to obtain *A* or *B*.	M1
	$A = 3,\ B = 1$	A1, A1 **(3)**
	If the cover-up rule is used, give M1 A1 for the first of *A* or *B* found, A1 for the second.	
	(b) $\int \frac{5x+3}{(2x-3)(x+2)}\,\mathrm{d}x = \frac{3}{2}\ln(2x-3) + \ln(x+2)$	M1 A1ft
	$\left[\ \dots\ \right]_2^6 = \frac{3}{2}\ln 9 + \ln 2$	M1 A1
	$= \ln 54$ cao	A1 **(5)** **[8]**

Question Number	Scheme		Marks
4.	$\int \frac{1}{(1-x^2)^{\frac{1}{2}}}\,dx = \int \frac{1}{(1-\sin^2\theta)^{\frac{3}{2}}}\cos\theta\,d\theta$ Use of $x = \sin\theta$ and $\frac{dx}{d\theta} = \cos\theta$		M1
	$= \int \frac{1}{\cos^2\theta}\,d\theta$		M1 A1
	$= \int \sec^2\theta\,d\theta = \tan\theta$		M1 A1
	Using the limits 0 and $\frac{\pi}{6}$ to evaluate integral		M1
	$[\tan\theta]_0^{\frac{\pi}{6}} = \frac{1}{\sqrt{3}} \quad \left(= \frac{\sqrt{3}}{3}\right)$	cao	A1
			[7]
	Alternative for final M1 A1		
	Returning to the variable x and using the limits 0 and $\frac{1}{2}$ to evaluate integral		M1
	$\left[\frac{x}{\sqrt{(1-x^2)}}\right]_0^{\frac{1}{2}} = \frac{1}{\sqrt{3}} \quad \left(= \frac{\sqrt{3}}{3}\right)$	cao	A1

Question Number	Scheme	Marks
5.	(a) $\int x e^{2x} dx = \frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x}\, dx$ Attempting parts in the right direction	M1 A1
	$= \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x}$	A1
	$\left[\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} \right]_0^1 = \frac{1}{4} + \frac{1}{4} e^2$	M1 A1
		(5)
	(b) $x = 0.4 \Rightarrow y \approx 0.89022$	
	$x = 0.8 \Rightarrow y \approx 3.96243$ Both are required to 5 d.p .	B1
		(1)
	(c) $I \approx \frac{1}{2} \times 0.2 \times [\ ...\ ]$	B1
	$\approx\ ...\ \times \left[0 + 7.38906 + 2\left(0.29836 + .89022 + 1.99207 + 3.96243 \right) \right]$	M1 A1ft
	ft their answers to (b)	
	$\approx 0.1 \times 21.67522$	
	≈ 2.168 cao	A1 **(4)**
		[10]
	Note $\frac{1}{4} + \frac{1}{4} e^2 \approx 2.097\ \ldots$	

Question Number	Scheme		Marks	
6.	(a) $\frac{dx}{dt} = -2\operatorname{cosec}^2 t,\ \frac{dy}{dt} = 4\sin t\cos t$	both	M1 A1	
	$\frac{dy}{dx} = \frac{-2\sin t\cos t}{\operatorname{cosec}^2 t}\quad \left(= -2\sin^3 t\cos t\right)$		M1 A1	
				(4)
	(b) At $t = \frac{\pi}{4}$, $x = 2, y = 1$	both x and y	B1	
	Substitutes $t = \frac{\pi}{4}$ into an attempt at $\frac{dy}{dx}$ to obtain gradient $\left(-\frac{1}{2}\right)$		M1	
	Equation of tangent is $y - 1 = -\frac{1}{2}(x - 2)$		M1 A1	
	Accept $x + 2y = 4$ or any correct equivalent			(4)
	(c) Uses $1 + \cot^2 t = \operatorname{cosec}^2 t$, or equivalent, to eliminate t		M1	
	$1 + \left(\frac{x}{2}\right)^2 = \frac{2}{y}$	correctly eliminates t	A1	
	$y = \frac{8}{4 + x^2}$	cao	A1	
	The domain is $x \ldots 0$		B1	(4) [12]
	An alternative in (c)			
	$\sin t = \left(\frac{y}{2}\right)^{\frac{1}{2}}$; $\cos t = \frac{x}{2}\sin t = \frac{x}{2}\left(\frac{y}{2}\right)^{\frac{1}{2}}$			
	$\sin^2 t + \cos^2 t = 1 \quad \Rightarrow \quad \frac{y}{2} + \frac{x^2}{4} \times \frac{y}{2} = 1$		M1 A1	
	Leading to $y = \frac{8}{4 + x^2}$		A1	

Question Number	Scheme	Marks
7.	(a) **k** component $2+4\lambda=-2 \Rightarrow \lambda=-1$	M1 A1
	Note $\mu=2$	
	Substituting their λ (or μ) into equation of line and obtaining B	M1
	B: $(2, 2, -2)$ Accept vector forms	A1
		(4)
	(b) $\left\lvert\begin{pmatrix}1\\-1\\4\end{pmatrix}\right\rvert=\sqrt{18};\quad\left\lvert\begin{pmatrix}1\\-1\\0\end{pmatrix}\right\rvert=\sqrt{2}$ both	B1
	$\begin{pmatrix}1\\-1\\4\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\0\end{pmatrix}=1+1+0\ (=2)$	B1
	$\cos\theta=\dfrac{2}{\sqrt{18}\sqrt{2}}=\dfrac{1}{3}$ cao	M1 A1
		(4)
	(c) $\overrightarrow{AB}=-\mathbf{i}+\mathbf{j}-4\mathbf{k}\Rightarrow\left\lvert\overrightarrow{AB}\right\rvert^2=18$ or $\left\lvert\overrightarrow{AB}\right\rvert=\sqrt{18}$ ignore direction of vector	M1
	$\overrightarrow{BC}=3\mathbf{i}-3\mathbf{j}\Rightarrow\left\lvert\overrightarrow{BC}\right\rvert^2=18$ or $\left\lvert\overrightarrow{BC}\right\rvert=\sqrt{18}$ ignore direction of vector	M1
	Hence $\left\lvert\overrightarrow{AB}\right\rvert=\left\lvert\overrightarrow{BC}\right\rvert$ *	A1 **(3)**
	(d) $\overrightarrow{OD}=6\mathbf{i}-2\mathbf{j}+2\mathbf{k}$ Allow first B1 for any two correct Accept column form or coordinates	B1 B1 **(2)** **[13]**

Question Number	Scheme	Marks
8.	(a) $\frac{dV}{dt}$ is the rate of increase of volume (with respect to time)	B1
	$-kV$: k is constant of proportionality and the negative shows decrease (or loss)	
	giving $\frac{dV}{dt} = 20 - kV$ ∗ These Bs are to be awarded independently	B1
		(2)
	(b) $\int \frac{1}{20-kV} dV = \int 1 dt$ separating variables	M1
	$-\frac{1}{k}\ln(20-kV) = t \quad (+C)$	M1 A1
	Using $V = 0, t = 0$ to evaluate the constant of integration	M1
	$c = -\frac{1}{k}\ln 20$	
	$t = \frac{1}{k}\ln\left(\frac{20}{20-kV}\right)$	
	Obtaining answer in the form $V = A + Be^{-kt}$	M1
	$V = \frac{20}{k} - \frac{20}{k}e^{-kt}$ Accept $\frac{20}{k}\left(1-e^{-kt}\right)$	A1 **(6)**
	(c) $\frac{dV}{dt} = 20e^{-kt}$ Can be implied	M1
	$\frac{dV}{dt} = 10, t = 5 \Rightarrow 10 = 20e^{-kt} \Rightarrow k = \frac{1}{5}\ln 2 \approx 0.139$	M1 A1
	At $t = 10$, $V = \frac{75}{\ln 2}$ awrt 108	M1 A1 **(5)**
		[13]
	Alternative to (b)	
	Using printed answer and differentiating $\frac{dV}{dt} = -kBe^{-kt}$	M1
	Substituting into differential equation	
	$-kBe^{-kt} = 20 - kA - kBe^{-kt}$	M1
	$A = \frac{20}{k}$	M1 A1
	Using $V = 0, t = 0$ in printed answer to obtain $A + B = 0$	M1
	$B = -\frac{20}{k}$	A1 **(6)**