

4. Use the substitution $x = \sin \theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}} dx.$$

(7)

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5.

Figure 1

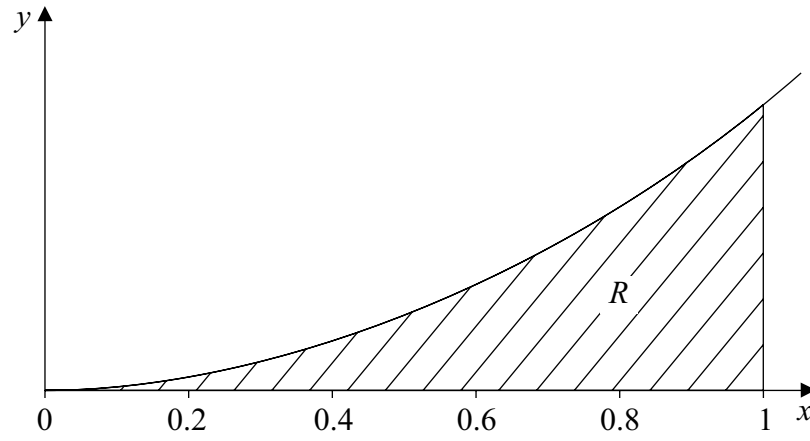


Figure 1 shows the graph of the curve with equation

$$y = xe^{2x}, \quad x \geq 0.$$

The finite region R bounded by the lines $x = 1$, the x -axis and the curve is shown shaded in Figure 1.

(a) Use integration to find the exact value for the area of R . (5)

(b) Complete the table with the values of y corresponding to $x = 0.4$ and 0.8 .

x	0	0.2	0.4	0.6	0.8	1
$y = xe^{2x}$	0	0.29836		1.99207		7.38906

(1)

(c) Use the trapezium rule with all the values in the table to find an approximate value for this area, giving your answer to 4 significant figures. (4)



Question 5 continued

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(Total 10 marks)

Q5	
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6. A curve has parametric equations

$$x = 2 \cot t, \quad y = 2 \sin^2 t, \quad 0 < t \leq \frac{\pi}{2}.$$

- (a) Find an expression for $\frac{dy}{dx}$ in terms of the parameter t . (4)

- (b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$. (4)

- (c) Find a cartesian equation of the curve in the form $y = f(x)$. State the domain on which the curve is defined. (4)



7. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix},$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point B and the acute angle between l_1 and l_2 is θ .

(a) Find the coordinates of B .

(4)

(b) Find the value of $\cos \theta$, giving your answer as a simplified fraction.

(4)

The point A , which lies on l_1 , has position vector $\mathbf{a} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

The point C , which lies on l_2 , has position vector $\mathbf{c} = 5\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.

The point D is such that $ABCD$ is a parallelogram.

(c) Show that $|\vec{AB}| = |\vec{BC}|$.

(3)

(d) Find the position vector of the point D .

(2)



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