

GCE

Edexcel GCE

Core Mathematics C3 (6665)

Summer 2005

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Mark Scheme (Results)

June 2005  
6665 Core C3  
Mark Scheme

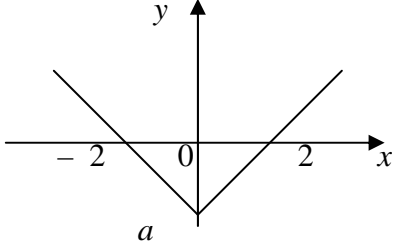
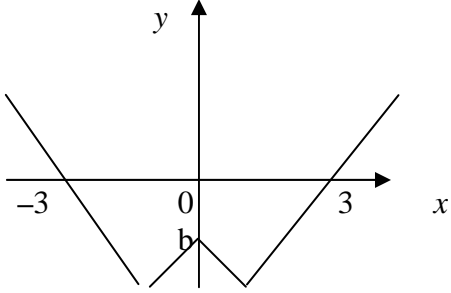
Question Number	Scheme	Marks
1. (a)	Dividing by $\cos^2 \theta$ : $\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} \equiv \frac{1}{\cos^2 \theta}$  Completion : $1 + \tan^2 \theta \equiv \sec^2 \theta$ (no errors seen)	M1  A1  (2)
(b)	Use of $1 + \tan^2 \theta = \sec^2 \theta$ : $2(\sec^2 \theta - 1) + \sec \theta = 1$ $[ 2\sec^2 \theta + \sec \theta - 3 = 0 ]$  Factorising or solving: $(2\sec \theta + 3)(\sec \theta - 1) = 0$ $[ \sec \theta = -\frac{3}{2} \text{ or } \sec \theta = 1 ]$ $\theta = 0$  $\cos \theta = -\frac{2}{3}$ ; $\theta_1 = 131.8^\circ$ $\theta_2 = 228.2^\circ$  $[A1ft \text{ for } \theta_2 = 360^\circ - \theta_1]$	M1  M1  B1  M1 A1 A1√  (6)  [ 8 ]

Question Number	Scheme	Marks
2. (a)	<p>(i) <math>6 \sin x \cos x + 2 \sec 2x \tan 2x</math>  <b>or</b> <math>3 \sin 2x + 2 \sec 2x \tan 2x</math> [M1 for <math>6 \sin x</math>]</p> <p>(ii) <math>3(x + \ln 2x)^2 \left(1 + \frac{1}{x}\right)</math>  [B1 for <math>3(x + \ln 2x)^2</math>]</p>	<p>M1A1A1 (3)</p> <p>B1M1A1 (3)</p>
(b)	<p>Differentiating numerator to obtain <math>10x - 10</math>  Differentiating denominator to obtain <math>2(x-1)</math></p> <p>Using quotient rule formula correctly:</p> <p>To obtain <math>\frac{dy}{dx} = \frac{(x-1)^2(10x-10) - (5x^2-10x+9)2(x-1)}{(x-1)^4}</math></p> <p>Simplifying to form <math>\frac{2(x-1)[5(x-1)^2 - (5x^2-10x+9)]}{(x-1)^4}</math></p> <p><math>= -\frac{8}{(x-1)^3}</math> * (c.s.o.)</p> <p><b>Alternatives for (b)</b>  <b>Either</b> Using product rule formula correctly:  Obtaining <math>10x - 10</math>  Obtaining <math>-2(x-1)^{-3}</math>  To obtain <math>\frac{dy}{dx} = (5x^2-10x+9)\{-2(x-1)^{-3}\} + (10x-10)(x-1)^{-2}</math></p> <p>Simplifying to form <math>\frac{10(x-1)^2 - 2(5x^2-10x+9)}{(x-1)^3}</math></p> <p><math>= -\frac{8}{(x-1)^3}</math> * (c.s.o.)</p> <p><b>Or</b> Splitting fraction to give <math>5 + \frac{4}{(x-1)^2}</math>  Then differentiating to give answer</p>	<p>B1 B1</p> <p>M1 A1</p> <p>M1 A1 (6)</p> <p>[12]</p> <p>M1 B1 B1 A1 cao</p> <p>M1 A1 (6)</p> <p>M1 B1 B1 M1 A1 A1 (6)</p>

Question Number	Scheme	Marks
3(a)	$\frac{5x + 1}{(x + 2)(x - 1)} - \frac{3}{x + 2}$ $= \frac{5x + 1 - 3(x - 1)}{(x + 2)(x - 1)}$ <p>M1 for combining fractions even if the denominator is not lowest common</p> $= \frac{2x + 4}{(x + 2)(x - 1)} = \frac{2(x + 2)}{(x + 2)(x - 1)} = \frac{2}{x - 1} \quad *$ <p>M1 must have linear numerator</p> <p>(b)</p> $y = \frac{2}{x - 1} \Rightarrow xy - y = 2 \Rightarrow xy = 2 + y$ $f^{-1}(x) = \frac{2 + x}{x} \quad \text{o.e.}$ <p>(c)</p> $fg(x) = \frac{2}{x^2 + 4} \quad (\text{attempt}) \quad \left[ \frac{2}{"g" - 1} \right]$ <p>Setting <math>\frac{2}{x^2 + 4} = \frac{1}{4}</math> and finding <math>x^2 = \dots; \quad x = \pm 2</math></p>	<p>B1</p> <p>M1</p> <p>M1 A1 cso</p> <p>(4)</p> <p>M1A1</p> <p>A1</p> <p>(3)</p> <p>M1</p> <p>M1; A1</p> <p>(3)</p> <p>[10]</p>

Question Number	Scheme	Marks
4	<p>(a) <math>f'(x) = 3e^x - \frac{1}{2x}</math></p> <p>(b) <math>3e^x - \frac{1}{2x} = 0</math></p> $\Rightarrow 6\alpha e^\alpha = 1 \quad \Rightarrow \alpha = \frac{1}{6} e^{-\alpha} \quad (*)$ <p>(c) <math>x_1 = 0.0613\dots, x_2 = 0.1568\dots, x_3 = 0.1425\dots, x_4 = 0.1445\dots</math></p> <p>[M1 at least <math>x_1</math> correct, A1 all correct to 4 d.p.]</p> <p>(d) Using <math>f'(x) = 3e^x - \frac{1}{2x}</math> with suitable interval</p> <p>e.g. <math>f'(0.14425) = -0.0007</math>  <math>f'(0.14435) = +0.002(1)</math></p> <p>Accuracy (change of sign and correct values)</p>	<p>M1A1A1 (3)</p> <p>M1</p> <p>A1 cso (2)</p> <p>M1 A1 (2)</p> <p>M1</p> <p>A1 (2)</p> <p>[9]</p>

Question Number	Scheme	Marks
5. (a)	$\cos 2A = \cos^2 A - \sin^2 A \quad (+ \text{ use of } \cos^2 A + \sin^2 A \equiv 1)$ $= (1 - \sin^2 A) - \sin^2 A = 1 - 2 \sin^2 A \quad (*)$	M1 A1 (2)
(b)	$2 \sin 2\theta - 3 \cos 2\theta - 3 \sin \theta + 3 \equiv 4 \sin \theta \cos \theta - 3(1 - 2 \sin^2 \theta) - 3 \sin \theta + 3$ $\equiv 4 \sin \theta \cos \theta + 6 \sin^2 \theta - 3 \sin \theta$ $\equiv \sin \theta (4 \cos \theta + 6 \sin \theta - 3) \quad (*)$	B1; M1 M1 A1 (4)
(c)	$4 \cos \theta + 6 \sin \theta \equiv R \sin \theta \cos \alpha + R \cos \theta \sin \alpha$ <p>Complete method for <math>R</math> (may be implied by correct answer)</p> $[ R^2 = 4^2 + 6^2, R \sin \alpha = 4, R \cos \alpha = 6 ]$ $R = \sqrt{52} \text{ or } 7.21$ <p>Complete method for <math>\alpha</math>; <math>\alpha = 0.588</math> (allow <math>33.7^\circ</math>)</p>	M1 A1 M1 A1 (4)
(d)	$\sin \theta (4 \cos \theta + 6 \sin \theta - 3) = 0$ $\theta = 0$ $\sin(\theta + 0.588) = \frac{3}{\sqrt{52}} = 0.4160.. \quad (24.6^\circ)$ $\theta + 0.588 = (0.4291), 2.7125 \text{ [or } \theta + 33.7^\circ = (24.6^\circ), 155.4^\circ]$ $\theta = 2.12 \text{ cao}$	M1 B1 M1 dM1 A1 (5)
		[15]

Question Number	Scheme	Marks
6. (a)	 <p style="text-align: center;"><math>y</math> <math>x</math> <math>a</math></p> <p>Translation ← by 1 Intercepts correct</p>	M1 A1 (2)
(b)	 <p style="text-align: center;"><math>y</math> <math>x</math> <math>b</math></p> <p><math>x \geq 0</math>, correct “shape” provided graph is not original graph Reflection in <math>y</math>-axis Intercepts correct</p>	B1 B1√ B1 (3)
(c)	$a = -2, b = -1$	B1B1 (2)
(d)	<p>Intersection of <math>y = 5x</math> with <math>y = -x - 1</math> Solving to give <math>x = -\frac{1}{6}</math></p> <p>[Notes: (i) If both values found for <math>5x = -x - 1</math> and <math>5x = x - 3</math>, or solved algebraically, can score 3 out of 4 for <math>x = -\frac{1}{6}</math> and <math>x = -\frac{3}{4}</math>; required to eliminate <math>x = -\frac{3}{4}</math> for final mark. (ii) Squaring approach: M1 correct method, <math>24x^2 + 22x + 3 = 0</math> ( correct 3 term quadratic, any form) A1 Solving M1, Final correct answer A1.]</p>	M1A1 M1A1 (4)
		<b>[11]</b>

7. (a)	<p>Setting <math>p = 300</math> at <math>t = 0 \Rightarrow 300 = \frac{2800a}{1 + a}</math></p> <p><math>(300 = 2500a); \quad a = 0.12</math> (c.s.o) *</p>	M1 dM1A1 (3)
(b)	<p><math>1850 = \frac{2800(0.12)e^{0.2t}}{1 + 0.12e^{0.2t}} ; \quad e^{0.2t} = 16.2\dots</math></p> <p>Correctly taking logs to <math>0.2 t = \ln k</math> <math>t = 14</math> (13.9..)</p>	M1A1 M1 A1 (4)
(c)	<p>Correct derivation: (Showing division of num. and den. by <math>e^{0.2t}</math> ; using <math>a</math>)</p>	B1 (1)
(d)	<p>Using <math>t \rightarrow \infty, e^{-0.2t} \rightarrow 0,</math></p> $p \rightarrow \frac{336}{0.12} = 2800$	M1 A1 (2) <b>[10]</b>