Paper Reference(s)

6687/01

Edexcel GCE

Statistics S5

Advanced Subsidiary

Thursday 9 June 2005 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Lilac)	Nil
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Statistics S5), the paper reference (6687), your surname, other name and signature.
Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. The random variable X has moment generating function

$$\mathrm{M}_X(t) = 1 + 13t + 99t^2 + 577t^3 + \ldots .$$

Find

(a) $\mathrm{E}(X)$,

(2)

(b) $\mathrm{Var}(X)$.

(3)

(Total 5 marks)

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2. Candidates for a certain examination have to sit two papers. Half of the candidates pass both papers, 75% pass paper I and 70% pass paper II. A pass in both papers secures a pass in the examination overall. In addition, 60% of those who pass paper I, but fail paper II, secure a pass in the examination overall, as do 40% of those who pass paper II but fail paper I.

A candidate for the examination is selected at random.

(a) Find the probability that this candidate secured a pass in the examination.

(6)

Given that this candidate passed the examination,

(b) find the probability that this candidate failed paper I or paper II.

(4)

(Total 10 marks)

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3. The random variable X has probability generating function

$$\mathrm{G}_X(t) = k[t^3(2 + 3t) + (1 + t)^4],$$

where k is a positive constant.

(a) Show that $k = \dfrac{1}{21}$.

(2)

Find

(b) $\mathrm{E}(X)$,

(3)

(c) $\mathrm{Var}(X)$,

(4)

(d) $\mathrm{P}(X = 3)$.

(2)

(Total 11 marks)

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4. Greg coughs at random and at a rate of 2 coughs every 5 minutes.

(a) Find the probability that Greg coughs 3 times in the next 5 minutes.

(2)

The random variable T represents the time in minutes between successive coughs.

(b) Find $\mathrm{P}(2 < T < 3)$.

(3)

Greg is accused of using his cough to send a secret code (2, 2) to Ruth. It is alleged that the number of complete minutes between the first and second coughs, and then between the second and third coughs, gives the code.

Assuming Greg continues to cough at random and at the same rate,

(c) find the probability that Ruth will interpret the times between his next two coughs as the code (2, 2).

(2)

(d) Comment on the accusation.

(1)

(Total 8 marks)

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5. The length of time it takes, in minutes, to serve a customer in Skipwood general stores is represented by the random variable X, which has an exponential distribution. The mean time taken is 2 minutes per customer.

(*a*) Write down the moment generating function of X.

(1)

The random variable $Y = X_1 + X_2 + X_3 + X_4$, where $X_i = 1, \ldots, 4$, are independent random variables, each having an exponential distribution with mean 2.

(*b*) Find the moment generating function of Y.

(2)

The random variable C has a χ^2-distribution with $2m$ degrees of freedom. The moment generating function of C is $\mathrm{M}_C(t) = \dfrac{1}{(1-2t)^m}$.

(*c*) Explain why Y has a χ^2-distribution and state its degrees of freedom.

(2)

One day Alan visits Skipwood general stores and finds a queue of 3 people in front of him. The person at the front of the queue is about to be served. The waiting times of customers are independent.

Find, to 2 decimal places, an approximation for the probability that Alan

(*d*) will leave the shop in less than $3\frac{1}{2}$ minutes,

(1)

(*e*) will be longer than 20 minutes in the shop.

(1)

(Total 7 marks)

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6. A child is repeatedly twisting a coloured spinner which has a probability 0.4 of landing on red. After each twist the child records whether or not the spinner lands on red.

(*a*) Show that the probability that the spinner lands on red for the first time occurs on or before the 7th twist is 0.972, to 3 decimal places.

(3)

Find the probability that

(*b*) exactly three reds occur during the first 7 twists,

(2)

(*c*) the 3rd red occurs on the 7th twist,

(3)

(*d*) the 3rd red occurs on or before the 7th twist.

(4)

On another occasion there are 3 children A, B and C playing with the spinner. The children take turns to twist the spinner. Child A starts, then B, then C, then A again and so on. The winner is the first child to have the spinner land on red.

(*e*) Find the probability that A wins.

(3)

Given that the first red occurs on or before the 7th twist,

(*f*) find the probability that A wins.

(2)

(Total 17 marks)

7. Components are delivered to a factory in large batches. The quality assurance manager uses the following double sampling plan for each batch delivered.

> A random sample of 10 components is examined and if there are fewer than 2 defective components the whole batch is accepted. If there are more than 2 defectives the entire batch is rejected. If exactly 2 of the components are defective then a second random sample of 10 components is examined. If there are no defective components in this second sample, then the whole batch is accepted; otherwise it is rejected.

(*a*) Find the probability of accepting a batch where the proportion of defective components is

(i) 10%, (ii) 15%.

(4)

(*b*) Show that, using this plan, the expected proportion of defective components actually accepted by the factory is less than 9% in both cases of part (*a*).

(3)

(*c*) Show that, using this plan, the expected number of items sampled, from a batch where the proportion of defectives is 10%, is approximately 12.

(2)

The quality assurance manager wants to set up a single sampling plan. The plan is to be based on a random sample of size 12. The probability of acceptance should be as high as possible but the expected proportion of defective components actually accepted by the factory must be less than 9%.

(*d*) Set up a suitable scheme for batches where the proportion of defectives is 10% and state the probability of acceptance.

(2)

The single sampling plan described in part (*d*) is used on a batch with 15% defective components.

(*e*) Find the expected proportion of defective components actually accepted by the factory in this case.

(2)

The manager decides to use the double sampling plan for all batches.

(*f*) Discuss the advantages and disadvantages of this decision for batches where the proportion of defective components is

(i) 10%, (ii) 15%.

(4)

(Total 17 marks)

TOTAL FOR PAPER: 75 MARKS

END