Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced/Advanced Subsidiary

Friday 24 June 2005 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Lilac or Green)	Nil
Answer Book (AB16)	
Graph paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take g = 9.8 m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has 7 questions.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. Two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are the only forces acting on a particle. $\mathbf{F}_1$ has magnitude 9 N and acts in the direction of $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$. $\mathbf{F}_2$ has magnitude 18 N and acts in the direction of $\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}$.

Find the total work done by the two forces in moving the particle from the point with position vector $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ m to the point with position vector $(3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ m.

(Total 6 marks)

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2. At time t seconds the position vector of a particle P, relative to a fixed origin O, is $\mathbf{r}$ metres, where $\mathbf{r}$ satisfies the differential equation

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + 2\mathbf{r} = 3\mathrm{e}^{-t}\mathbf{j}.$$

Given that $\mathbf{r} = 2\mathbf{i} - \mathbf{j}$ when $t = 0$, find $\mathbf{r}$ in terms of t.

(Total 7 marks)

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3. A system of forces acting on a rigid body consists of two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting at a point A of the body, together with a couple of moment $\mathbf{G}$. $\mathbf{F}_1 = (\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ N and $\mathbf{F}_2 = (-2\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ N. The position vector of the point A is $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ m and $\mathbf{G} = (7\mathbf{i} - 3\mathbf{j} + 8\mathbf{k})$ Nm.

Given that the system is equivalent to a single force $\mathbf{R}$,

(*a*) find $\mathbf{R}$,

(2)

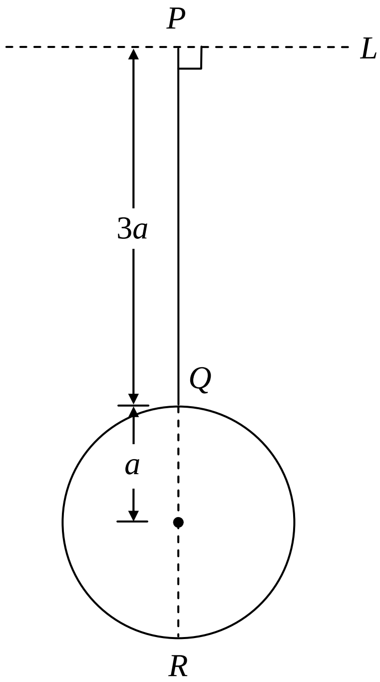
(*b*) find a vector equation for the line of action of $\mathbf{R}$.

(7)

(Total 9 marks)

4.

Figure 1



A thin uniform rod PQ has mass m and length $3a$. A thin uniform circular disc, of mass m and radius a, is attached to the rod at Q in such a way that the rod and the diameter QR of the disc are in a straight line with $PR = 5a$. The rod together with the disc form a composite body, as shown in Figure 1. The body is free to rotate about a fixed smooth horizontal axis L through P, perpendicular to PQ and in the plane of the disc.

(a) Show that the moment of inertia of the body about L is $\dfrac{77ma^2}{4}$.

(7)

When PR is vertical, the body has angular speed ω and the centre of the disc strikes a stationary particle of mass $\frac{1}{2}m$. Given that the particle adheres to the centre of the disc,

(b) find, in terms of ω, the angular speed of the body immediately after the impact.

(4)

(Total 11 marks)

5. A uniform square lamina *ABCD*, of mass m and side $2a$, is free to rotate in a vertical plane about a fixed smooth horizontal axis L which passes through A and is perpendicular to the plane of the lamina. The moment of inertia of the lamina about L is $\frac{8ma^2}{3}$.

Given that the lamina is released from rest when the line AC makes an angle of $\frac{\pi}{3}$ with the downward vertical,

(*a*) find the magnitude of the vertical component of the force acting on the lamina at A when the line AC is vertical.

(7)

Given instead that the lamina now makes small oscillations about its position of stable equilibrium,

(*b*) find the period of these oscillations.

(5)

(Total 12 marks)

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6. A rocket-driven car moves along a straight horizontal road. The car has total initial mass M. It propels itself forwards by ejecting mass backwards at a constant rate λ per unit time at a constant speed U relative to the car. The car starts from rest at time $t = 0$. At time t the speed of the car is v. The total resistance to motion is modelled as having magnitude kv, where k is a constant.

Given that $t < \frac{M}{\lambda}$, show that

(*a*) $\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\lambda U - kv}{M - \lambda t}$,

(7)

(*b*) $v = \frac{\lambda U}{k}\left\{1 - \left(1 - \frac{\lambda t}{M}\right)^{\frac{k}{\lambda}}\right\}$.

(6)

(Total 13 marks)

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7. A uniform lamina of mass m is in the shape of an equilateral triangle ABC of perpendicular height h. The lamina is free to rotate in a vertical plane about a fixed smooth horizontal axis L through A and perpendicular to the lamina.

(*a*) Show, by integration, that the moment of inertia of the lamina about L is $\dfrac{5mh^2}{9}$.

(9)

The centre of mass of the lamina is G. The lamina is in equilibrium, with G below A, when it is given an angular speed $\sqrt{\left(\dfrac{6g}{5h}\right)}$.

(*b*) Find the angle between AG and the downward vertical, when the lamina first comes to rest.

(5)

(*c*) Find the greatest magnitude of the angular acceleration during the motion.

(3)

(Total 17 marks)

TOTAL FOR PAPER: 75 MARKS

END