Edexcel GCE

Pure Mathematics P6
Further Pure Mathematics FP3
Advanced/Advanced Subsidiary

Tuesday 28 June 2005 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Lilac)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P6), the paper reference (6676), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has 7 questions.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. (*a*) Express $\dfrac{6x+10}{x+3}$ in the form $p + \dfrac{q}{x+3}$, where p and q are integers to be found.

(1)

The sequence of real numbers u_1, u_2, u_3, ... is such that $u_1 = 5.2$ and $u_{n+1} = \dfrac{6u_n + 10}{u_n + 3}$.

(*b*) Prove by induction that $u_n > 5$, for $n \in \mathbb{Z}^+$.

(4)

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2. (*a*) (i) Explain why, for any two vectors **a** and **b**, $\mathbf{a.b} \times \mathbf{a} = 0$.

(2)

(ii) Given vectors **a**, **b** and **c** such that $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, where $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{b} \neq \mathbf{c}$, show that

$$\mathbf{b} - \mathbf{c} = \lambda\mathbf{a}, \quad \text{where } \lambda \text{ is a scalar.}$$

(2)

(*b*) **A**, **B** and **C** are 2×2 matrices.

(i) Given that $\mathbf{AB} = \mathbf{AC}$, and that **A** is not singular, prove that $\mathbf{B} = \mathbf{C}$.

(2)

(ii) Given that $\mathbf{AB} = \mathbf{AC}$, where $\mathbf{A} = \begin{pmatrix} 3 & 6 \\ 1 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}$, find a matrix **C** whose elements are all non-zero.

(3)

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3. The line l_1 has equation

$$\mathbf{r} = \mathbf{i} + 6\mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{k})$$

and the line l_2 has equation

$$\mathbf{r} = 3\mathbf{i} + p\mathbf{j} + \mu(\mathbf{i} - 2\mathbf{j} + \mathbf{k}), \text{ where } p \text{ is a constant.}$$

The plane Π_1 contains l_1 and l_2.

(a) Find a vector which is normal to Π_1.

(2)

(b) Show that an equation for Π_1 is $6x + y - 4z = 16$.

(2)

(c) Find the value of p.

(1)

The plane Π_2 has equation $\mathbf{r}.(\mathbf{i} + 2\mathbf{j} + \mathbf{k}) = 2$.

(d) Find an equation for the line of intersection of Π_1 and Π_2, giving your answer in the form

$$(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}.$$

(5)

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4. A complex number z is represented by the point P in the Argand diagram. Given that

$$\left| z - 3\mathrm{i} \right| = 3,$$

(a) sketch the locus of P.

(2)

(b) Find the complex number z which satisfies both $\left| z - 3\mathrm{i} \right| = 3$ and $\arg(z - 3\mathrm{i}) = \frac{3}{4}\pi$.

(4)

The transformation T from the z-plane to the w-plane is given by

$$w = \frac{2\mathrm{i}}{w}.$$

(c) Show that T maps $\left| z - 3\mathrm{i} \right| = 3$ to a line in the w-plane, and give the cartesian equation of this line.

(5)

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5. (a) Given that $z = e^{i\theta}$, show that

$$z^n - \frac{1}{z^n} = 2i \sin n\theta,$$

where n is a positive integer.

(2)

(b) Show that

$$\sin^5 \theta = \frac{1}{16}(\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta).$$

(5)

(c) Hence solve, in the interval $0 \le \theta < 2\pi$,

$$\sin 5\theta - 5 \sin 3\theta + 6 \sin \theta = 0.$$

(5)

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6. The variable y satisfies the differential equation

$$4(1 + x^2)\frac{d^2 y}{dx^2} + 4x\frac{dy}{dx} = y.$$

At $x = 0$, $y = 1$ and $\dfrac{dy}{dx} = \dfrac{1}{2}$.

(a) Find the value of $\dfrac{d^2 y}{dx^2}$ at $x = 0$.

(1)

(b) Use the approximations

$$\left(\frac{d^2 y}{dx^2}\right)_0 \approx \frac{(y_1 - 2y_0 + y_{-1})}{h^2} \quad \text{and} \quad \left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_{-1}}{2h}, \text{ with } h = 0.1,$$

to find an estimate of y, at $x = 0.1$, to 5 decimal places.

(6)

(c) Find the value of $\dfrac{d^3 y}{dx^3}$ at $x = 0$.

(4)

(d) Express y as a series, in ascending powers of x, up to and including the term in x^3.

(2)

(e) Find the value that the series gives for y at $x = 0.1$, giving your answer to 5 decimal places.

(1)

7.
$$\mathbf{A} = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & k \end{pmatrix}.$$

(*a*) Show that det $\mathbf{A} = 20 - 4k$.

(2)

(*b*) Find $\mathbf{A}^{-1}$.

(6)

Given that $k = 3$ and that $\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$,

(*c*) find the corresponding eigenvalue.

(2)

Given that the only other distinct eigenvalue of $\mathbf{A}$ is 8,

(*d*) find a corresponding eigenvector.

(4)

TOTAL FOR PAPER: 75 MARKS

END