| Question Number | Scheme | Marks |
|---|---|---|
| **1.** (*a*) | P(*B*) $= P(B \cap D) + P(B \cap F) + P(B \cap R)$ | |
| | $= 0.3 \times 0.8 + 0.6 \times 0.6 + 0.1 \times 0.1$ | M1 |
| | $= 0.61$ | A1 (2) |
| (*b*) | $P(R \mid B) = \dfrac{P(R \cap B)}{P(B)} =, \dfrac{0.01}{0.61} = \dfrac{1}{61}$ (accept 0.016 or 0.0164) | M1, A1 (2) |
| (*c*) | $\dfrac{1}{61} + \dfrac{1}{61} - \dfrac{1}{61}^2$ or $1 - \left(1 - \dfrac{1}{61}\right)^2$ | M1 M1 |
| | $= 0.0325$ awrt 0.0325 | A1 |
| | | **(7 marks)** |
| **2.** (*a*) | $f(x) = \begin{cases} \frac{3}{7}e^{-\frac{3}{7}x} & [x \geq 0] \\ [0 & \text{otherwise}] \end{cases}$ | M1 A1 (2) |
| (*b*) | $P(2 < X < 3) = \int_2^3 \frac{3}{7}e^{-\frac{3}{7}x}\,dx$ | M1 |
| | $= \left[-e^{-\frac{3}{7}x}\right]_2^3$ | A1 |
| | $= e^{-\frac{6}{7}} - e^{-\frac{9}{7}} = 0.14791$ awrt 0.148 | A1 (3) |
| (*c*) | $P(X \geq 7) = \left[-e^{-\frac{3}{7}x}\right]_7^\infty$ | M1 |
| | $= e^{-3} = 0.049787$ awrt 0.050 | A1 (2) |
| | | **(7 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **3.** (*a*) | Geometric | B1 (1) |
| (*b*) | $p = \frac{1}{8}$ | B1 |
| | $P(S = 5) = \left(\frac{7}{8}\right)^4 \times \left(\frac{1}{8}\right)$ | M1 |
| | $= 0.073$ | A1 (3) |
| (*c*) | $P(S \geq 3) = (1 - p)^2$ | M1 A1 ft |
| | $= \left(\frac{7}{8}\right)^2 = \frac{49}{64}$ awrt 0.766 | A1 (3) |
| (*d*) | Assume shots are *independent* and *probability* of hits is *constant* | B1 B1 (2) |
| | | **(9 marks)** |
| **4.** (*a*) | $M_X(t) = E(e^{tx}) = \int_0^a e^{tx} \frac{1}{a} dx$ | M1 |
| | $= \left[\frac{1}{at} e^{tx}\right]_0^a$ | M1 |
| | $= \left(\frac{e^{at}}{at}\right) - \left(\frac{1}{at}\right)$ | M1 |
| | $= \frac{e^{at} - 1}{at}$ | A1 cso (4) |
| (*b*) | $M_Y(0) - 1 \Rightarrow 1 = \frac{1}{4}(1 + A + B)$ or $A + B = 3$ (1) | M1 |
| | $M'_Y(t) = \frac{1}{4}(Ae^t + 2Be^{2t})$ | M1 |
| | $E(Y) = M'_Y(0) \Rightarrow \frac{5}{4} = \frac{A}{8} + \frac{2B}{4}$ or $A + 2B = 5$ (2) | A1 |
| | (2) – (1) $\Rightarrow$ $B = 2$ and $A = 1$ | M1 A1 A1 (6) |
| (*c*) | $M_Z(t) = M_X(t) \times M_Y(t) = \frac{e^{at} - 1}{at} \times \frac{1 + e^t + 2e^{2t}}{4}$ | B1 ft (1) |
| | | **(11 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **5.** (*a*) | $60 = \frac{3(1-p)}{p^2}$ | M1 A1 |
| | $20p = 1 - p$ | M1 |
| | $(5p - 1)(4p + 1) = 0$ | M1 |
| | $p = \frac{1}{5}$ | A1 (5) |
| (*b*) | $P(Y = 8) = \binom{7}{2} p^2 (1-p)^5 \times p$ | M1 |
| | $= \binom{7}{2} \times \left(\frac{1}{5}\right)^3 \times \left(\frac{4}{5}\right)^5 = 0.05505$ awrt 0.055 | A1 (2) |
| (*c*) | $P(Y \le 10 \mid ...) = 1 - P(Y \ge 11 \mid \text{1st head on 2nd toss})$ | M1 |
| | $= 1 - P(\text{0 heads in 8 tosses}) - P(\text{1 head in 8 tosses})$ | M1 |
| | $= 1 - 0.8^8 - 8 \times 0.2 \times (0.8)^7$ | A1 A1 |
| | $= 0.49688$ awrt 0.497 | A1 (5) |
| | | **(12 marks)** |
| **6.** (*a*) | $P(\text{Accept}) = P(X \le 1 \mid X \sim B(20, p))$ | M1 |
| | $= (1-p)^{20} + 20(1-p)^{19}p$ | M1 |
| | $= (1-p)^{19}(1 + 19p)$ (*) | A1 cso (3) |
| (*b*) | $j = 0.880, \quad k = 0.587$ | B1, B1 (2) |
| (*c*) | Graph — axes and scales | B1 |
| | points | B1 |
| | OC curve | B1 (3) |
| (*d*)(i) | $p = 0.015 \Rightarrow P = 0.96 \Rightarrow P(\text{Reject}) = 0.04$ | B1 |
| (ii) | $p = 0.065 \Rightarrow P = 0.62 \Rightarrow P(\text{Reject}) = 0.38$ | M1 A1 (3) |
| (*e*) | High *probability of acceptance for low p is OK* | |
| | but not *very efficient* since *negative gradient is not steep enough* | B1 B1 (2) |
| | | **(13 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **7.** (*a*)(i) | $P(X = 2) = \dfrac{e^{-3} \times 3^2}{2!} = 4.5e^{-3}$ | M1 A1 |
| (ii) | $P(X \geq 4) = 1 - P(X \leq 3), \quad = 1 - e^{-3}\left(1 + 3 + \dfrac{3^2}{2!} + \dfrac{3^3}{3!}\right)$ | M1, A1 |
| | $= 1 - 13e^{-3}$ | A1 (5) |
| (*b*) | $y$: 0 1 2 3 4 | |
| | $x$: 0 1 2 3 $\geq 4$ | |
| | $P(Y = y)$: $e^{-3}$ $3e^{-3}$ $4.5e^{-3}$ $4.5e^{-3}$ $1 - 13e^{-3}$ | B1 |
| | $G_Y(t) = e^{-3}(t^0 + 3t + 4.5t^2 + 4.5t^3) + (1 - 13e^{-3})t^4$ | M1 |
| | $= e^{-3}(1 + 3t + 4.5t^2 + 4.5t^3 - 13t^4$ (*) | A1 cso (3) |
| (*c*) | $G'_Y(t) = e^{-3}(3 + 9t + 13.5t^2 - 52t^3 + 4t^4$ | M1 A1 |
| | $\mu = E(Y) = G'_Y(1) = 4 - 26.5e^{-3}$ or 2.68 | A1 |
| | $G''_Y(t) = e^{-3}(9 + 27t - 156t^2) + 12t^2$ | M1 A1 |
| | $G''_Y(1) = e^{-3}(-120) + 12 = 12 - 120e^{-3}$ | A1 |
| | $\sigma^2 = G''_Y(1) + G'_Y(1) - [G'_Y(1)]^2 \quad (= 1.52...)$ | M1 |
| | $\sigma = \sqrt{\sigma^2} = 1.23$ | A1(8) |
| | | **(15 marks)** |