Paper Reference(s)

6673

Edexcel GCE

Pure Mathematics P3

Advanced/Advanced Subsidiary

Wednesday 9 June 2004 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16) Graph Paper (ASG2) Mathematical Formulae (Lilac)	Nil

Candidates may only use one of the basic scientific calculators approved by the Qualifications and Curriculum Authority.

Instructions to Candidates

In the boxes on the Answer Book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P3), the paper reference (6673), your surname, other names and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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Turn over

1. Use the derivatives of $\operatorname{cosec} x$ and $\cot x$ to prove that

$$\frac{d}{dx}[\ln(\operatorname{cosec} x + \cot x)] = -\operatorname{cosec} x.$$

(3)

2.

$$f(x) = (x^2 + p)(2x + 3) + 3,$$

where p is a constant.

(a) Write down the remainder when $f(x)$ is divided by $(2x + 3)$.

(1)

Given that the remainder when $f(x)$ is divided by $(x - 2)$ is 24,

(b) prove that $p = -1$,

(2)

(c) factorise $f(x)$ completely.

(4)

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3. The circle C has centre (5, 13) and touches the x-axis.

(a) Find an equation of C in terms of x and y.

(2)

(b) Find an equation of the tangent to C at the point (10, 1), giving your answer in the form $ay + bx + c = 0$, where a, b and c are integers.

(5)

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4. Use the substitution $u = 1 + \sin x$ and integration to show that

$$\int \sin x \cos x (1 + \sin x)^5 \, dx = \frac{1}{42}(1 + \sin x)^6 [6 \sin x - 1] + \text{constant}.$$

(8)

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5. Given that

$$\frac{3+5x}{(1+3x)(1-x)} \equiv \frac{A}{1+3x} + \frac{B}{1-x},$$

(a) find the values of the constants A and B.

(3)

(b) Hence, or otherwise, find the series expansion in ascending powers of x, up to and including the term in x^2, of

$$\frac{3+5x}{(1+3x)(1-x)}.$$

(5)

(c) State, with a reason, whether your series expansion in part (b) is valid for $x = \frac{1}{2}$.

(2)

6.

Figure 1

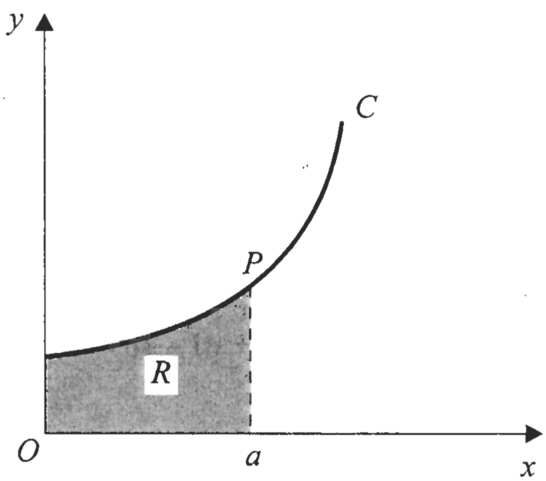

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 3t\sin t, \quad y = 2\sec t, \quad 0 \leqslant t < \frac{\pi}{2}.$$

The point $P(a, 4)$ lies on C.

(a) Find the exact value of a.

(3)

The region R is enclosed by C, the axes and the line $x = a$, as shown in Fig. 1.

(b) Show that the area of R is given by

$$6\int_0^{\frac{\pi}{3}} (\tan t + t)\,dt.$$

(4)

(c) Find the exact value of the area of R.

(4)

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7. A drop of oil is modelled as a circle of radius r. At time t

$$r = 4(1 - e^{-\lambda t}), \qquad t > 0,$$

where λ is a positive constant.

(a) Show that the area A of the circle satisfies

$$\frac{dA}{dt} = 32\pi\lambda(e^{-\lambda t} - e^{-2\lambda t}).$$

(5)

In an alternative model of the drop of oil its area A at time t satisfies

$$\frac{dA}{dt} = \frac{A^{\frac{3}{2}}}{t^2}, \qquad t > 0.$$

Given that the area of the drop is 1 at $t = 1$,

(b) find an expression for A in terms of t for this alternative model.

(7)

(c) Show that, in the alternative model, the value of A cannot exceed 4.

(1)

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8. Relative to a fixed origin O, the vector equations of the two lines l_1 and l_2 are

$$l_1\text{: } \mathbf{r} = 9\mathbf{i} + 2\mathbf{j} + 4\mathbf{k} + t(-8\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}),$$

and

$$l_2\text{: } \mathbf{r} = -16\mathbf{i} + \alpha\mathbf{j} + 10\mathbf{k} + s(\mathbf{i} - 4\mathbf{j} + 9\mathbf{k}),$$

where α is a constant.

The two lines intersect at the point A.

(a) Find the value of α.

(6)

(b) Find the position vector of the point A.

(1)

(c) Prove that the acute angle between l_1 and l_2 is 60°.

(5)

Point B lies on l_1 and point C lies on l_2. The triangle ABC is equilateral with sides of length $14\sqrt{2}$.

(d) Find one of the possible position vectors for the point B and the corresponding position vector for the point C.

(4)

END