Paper Reference(s)

# 6681

# Edexcel GCE

## Mechanics M5

## Advanced/Advanced Subsidiary

## Friday 25 June 2004 – Morning

## Time: 1 hour 30 minutes

| **Materials required for examination** | **Items included with question papers** |
|---|---|
| Answer Book (AB16) | Nil |
| Graph Paper (ASG2) | |
| Mathematical Formulae (Lilac) | |

**Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.**

### Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of $g$ is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has six questions.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a rigid body. $\mathbf{F}_1 = (12\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})$ N and acts at the point with position vector $(2\mathbf{i} - 3\mathbf{j})$ m, $\mathbf{F}_2 = (-3\mathbf{j} + 2\mathbf{k})$ N and acts at the point with position vector $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ m. The force $\mathbf{F}_3$ acts at the point with position vector $(2\mathbf{i} - \mathbf{k})$ m.

Given that this set of forces is equivalent to a couple, find

(*a*) $\mathbf{F}_3$,

**(2)**

(*b*) the magnitude of the couple.

**(5)**

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2. Two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are the only forces acting on a particle $P$ of mass 2 kg. The particle is initially at rest at the point $A$ with position vector $(-2\mathbf{i} - \mathbf{j} - 4\mathbf{k})$ m. Four seconds later, $P$ is at the point $B$ with position vector $(6\mathbf{i} - 5\mathbf{j} + 8\mathbf{k})$ m.

Given that $\mathbf{F}_1 = (12\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})$ N, find

(*a*) $\mathbf{F}_2$,

**(5)**

(*b*) the work done on $P$ as it moves from $A$ to $B$.

**(3)**

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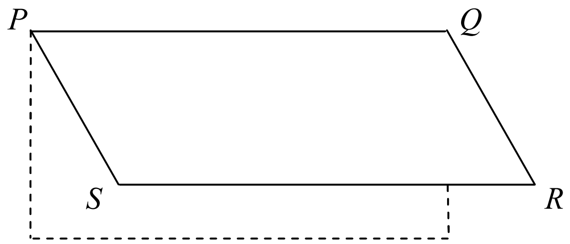
**3.** A uniform lamina of mass $m$ is in the shape of a rectangle $PQRS$, where $PQ = 8a$ and $QR = 6a$.

(*a*) Find the moment of inertia of the lamina about the edge $PQ$.

**(2)**

**Figure 1**




The flap on a letterbox is modelled as such a lamina. The flap is free to rotate about an axis along its horizontal edge $PQ$, as shown in Fig. 1. The flap is released from rest in a horizontal position. It then swings down into a vertical position.

(*b*) Show that the angular speed of the flap as it reaches the vertical position is $\sqrt{\left(\frac{g}{2a}\right)}$.

**(3)**

(*c*) Find the magnitude of the vertical component of the resultant force of the axis $PQ$ on the flap, as it reaches the vertical position.

**(4)**

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**4.** A uniform circular disc, of mass $m$ and radius $r$, has a diameter $AB$. The point $C$ on $AB$ is such that $AC = \frac{1}{2}r$. The disc can rotate freely in a vertical plane about a horizontal axis through $C$, perpendicular to the plane of the disc. The disc makes small oscillations in a vertical plane about the position of equilibrium in which $B$ is below $A$.

(*a*) Show that the motion is approximately simple harmonic.

**(6)**

(*b*) Show that the period of this approximate simple harmonic motion is $\pi\sqrt{\left(\frac{6r}{g}\right)}$.

**(1)**

The speed of $B$ when it is vertically below $A$ is $\sqrt{\left(\frac{gr}{54}\right)}$. The disc comes to rest when $CB$ makes an angle $\alpha$ with the downward vertical.

(*c*) Find an approximate value of $\alpha$.

**(3)**

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**5.** A rocket is launched vertically upwards under gravity from rest at time $t = 0$. The rocket propels itself upward by ejecting burnt fuel vertically downwards at a constant speed $u$ relative to the rocket. The initial mass of the rocket, including fuel, is $M$. At time $t$, before all the fuel has been used up, the mass of the rocket, including fuel, is $M(1 - kt)$ and the speed of the rocket is $v$.

(*a*) Show that $\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{ku}{1 - kt} - g$.

**(7)**

(*b*) Hence find the speed of the rocket when $t = \dfrac{1}{3k}$.

**(3)**

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**6.** A particle $P$ of mass 2 kg moves in the $x$-$y$ plane. At time $t$ seconds its position vector is $\mathbf{r}$ metres. When $t = 0$, the position vector of $P$ is $\mathbf{i}$ metres and the velocity of $P$ is $(-\mathbf{i} + \mathbf{j})$ m s$^{-1}$.

The vector $\mathbf{r}$ satisfies the differential equation

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} + 2\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + 2\mathbf{r} = \mathbf{0}.$$

(*a*) Find $\mathbf{r}$ in terms of $t$.

**(8)**

(*b*) Show that the speed of $P$ at time $t$ is $\mathrm{e}^{-t}\sqrt{2}$ m s$^{-1}$.

**(5)**

(*c*) Find, in terms of e, the loss of kinetic energy of $P$ in the interval $t = 0$ to $t = 1$.

**(2)**

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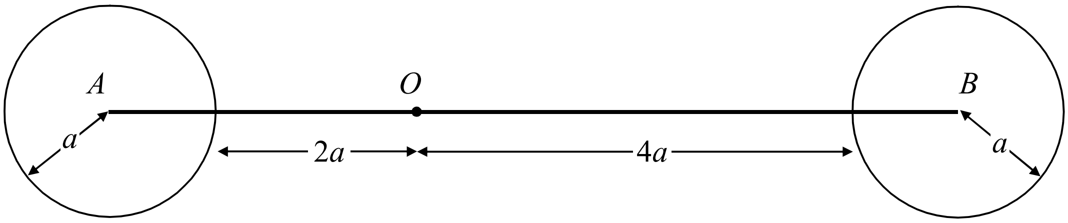
**7.**

**Figure 2**




A body consists of two uniform circular discs, each of mass $m$ and radius $a$, with a uniform rod. The centres of the discs are fixed to the ends $A$ and $B$ of the rod, which has mass $3m$ and length $8a$. The discs and the rod are coplanar, as shown in Fig. 2. The body is free to rotate in a vertical plane about a smooth fixed horizontal axis. The axis is perpendicular to the plane of the discs and passes through the point $O$ of the rod, where $AO = 3a$.

($a$) Show that the moment of inertia of the body about the axis is $54ma^2$.

**(6)**

The body is held at rest with $AB$ horizontal and is then released. When the body has turned through an angle of 30°, the rod $AB$ strikes a small fixed smooth peg $P$ where $OP = 3a$. Given that the body rebounds from the peg with its angular speed halved by the impact,

($b$) show that the magnitude of the impulse exerted on the body by the peg at the impact is $9m\sqrt{\left(\frac{5ga}{6}\right)}$.

**(10)**

**END**