Paper Reference(s)

6676

Edexcel GCE

Pure Mathematics P6

Advanced/Advanced Subsidiary

Tuesday 29 June 2004 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P6), the paper reference (6676), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. $$\frac{dy}{dx} + \frac{1}{10}y^2 = x, \qquad y = 2 \text{ at } x = 1.$$

Use the approximation $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$, with a step length of 0.1 to estimate the values of y at $x = 1.1$ and $x = 1.2$, giving your answers to 2 decimal places.

(6)

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2. Given that $y = \tan x$,

(a) find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$.

(3)

(b) Find the Taylor series expansion of $\tan x$ in ascending powers of $\left(x - \frac{\pi}{4}\right)$ up to and including the term in $\left(x - \frac{\pi}{4}\right)^3$.

(3)

(c) Hence show that $\tan \frac{3\pi}{10} \approx 1 + \frac{\pi}{10} + \frac{\pi^2}{200} + \frac{\pi^3}{3000}$.

(2)

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3. The points A, B and C lie on the plane Π and, relative to a fixed origin O, they have position vectors

$$\mathbf{a} = 3\mathbf{i} - \mathbf{j} + 4\mathbf{k}, \quad \mathbf{b} = -\mathbf{i} + 2\mathbf{j}, \quad \mathbf{c} = 5\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$$

respectively.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$.

(4)

(b) Find an equation of Π in the form $\mathbf{r.n} = p$.

(2)

The point D has position vector $5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

(c) Calculate the volume of the tetrahedron $ABCD$.

(4)

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4. (a) Prove by induction that

$$\frac{d^n}{dx^n}(e^x \cos x) = 2^{\frac{1}{2}n} e^x \cos\left(x + \tfrac{1}{4}n\pi\right), \qquad n \geq 1.$$

(8)

(b) Hence find the Maclaurin series expansion of $e^x \cos x$, in ascending powers of x, up to and including the term in x^4.

(3)

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5. The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 4 & -1 \\ 3 & 0 & p \\ a & b & c \end{pmatrix},$$

where p, a, b and c are constants and $a > 0$.

Given that $\mathbf{M}\mathbf{M}^{\mathrm{T}} = k\mathbf{I}$ for some constant k, find

(*a*) the value of p,

(2)

(*b*) the value of k,

(2)

(*c*) the values of a, b and c,

(6)

(*d*) $|\det \mathbf{M}|$.

(2)

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6. The transformation R is represented by the matrix **A**, where

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}.$$

(*a*) Find the eigenvectors of **A**.

(5)

(*b*) Find an orthogonal matrix **P** and a diagonal matrix **D** such that

$$\mathbf{A} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}.$$

(5)

(*c*) Hence describe the transformation R as a combination of geometrical transformations, stating clearly their order.

(4)

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7. The transformation T from the complex z-plane to the complex w-plane is given by

$$w = \frac{z+1}{z+\mathrm{i}}, \qquad z \neq -\mathrm{i}.$$

(*a*) Show that T maps points on the half-line $\arg(z) = \frac{\pi}{4}$ in the z-plane into points on the circle $|w| = 1$ in the w-plane.

(4)

(*b*) Find the image under T in the w-plane of the circle $|z| = 1$ in the z-plane.

(6)

(*c*) Sketch on separate diagrams the circle $|z| = 1$ in the z-plane and its image under T in the w-plane.

(2)

(*d*) Mark on your sketches the point P, where $z = \mathrm{i}$, and its image Q under T in the w-plane.

(2)

END