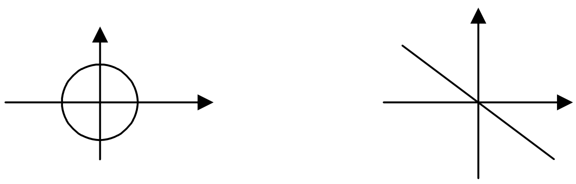
Question number	Scheme	Marks
1.	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = x_0 - \frac{1}{10}y_0^2 = 1 - 0.4 \quad (= 0.6) \quad$ (Possibly implicit)	B1
	$y_1 = 0.1\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 + y_0 = (0.1 \times 0.6) + 2 = 2.06$	M1 A1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1 = x_1 - \frac{1}{10}y_1^2 = 1.1 - \frac{1}{10}(2.06)^2 \quad (= 0.67564)$	A1ft
	$y_2 = 0.1\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1 + y_1 = 0.067564 + 2.06 = 2.13 \quad$ (2 d.p.)	M1 A1
		6
2.	(a) $\mathrm{f}'(x) = \sec^2 x \qquad \mathrm{f}''(x) = 2\sec x(\sec x \tan x) \quad$ (or equiv.)	M1 A1
	$\mathrm{f}'''(x) = 2\sec^2 x(\sec^2 x) + 2\tan x(2\sec^2 x \tan x) \quad$ (or equiv.)	A1 (3)
	$(2\sec^2 x + 6\sec^2 x \tan^2 x)$	
	$(2\sec^4 x + 4\sec^2 x \tan^2 x)$, $(6\sec^4 x - 4\sec^2 x)$, $(2 + 8\tan^2 x + 6\tan^4 x)$	
	(b) $\tan\frac{\pi}{4} = 1$ or $\sec\frac{\pi}{4} = \sqrt{2} \qquad$ (1, 2, 4, 16)	B1
	$\tan x = \mathrm{f}\left(\frac{\pi}{4}\right) + \left(x - \frac{\pi}{4}\right)\mathrm{f}'\left(\frac{\pi}{4}\right) + \frac{1}{2}\left(x - \frac{\pi}{4}\right)^2 \mathrm{f}''\left(\frac{\pi}{4}\right) + \frac{1}{6}\left(x - \frac{\pi}{4}\right)^3 \mathrm{f}'''\left(\frac{\pi}{4}\right)$	M1
	$= 1 + 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 + \frac{8}{3}\left(x - \frac{\pi}{4}\right)^3 \quad$ (Allow equiv. fractions)	A1(cso) (3)
	(c) $x = \frac{3\pi}{10}$, so use $\left(\frac{3\pi}{10} - \frac{\pi}{4}\right) \qquad \left(= \frac{\pi}{20}\right)$	M1
	$\tan\frac{3\pi}{10} \approx 1 + \frac{\pi}{10} + \left(2 \times \frac{\pi^2}{400}\right) + \left(\frac{8}{3} \times \frac{\pi^3}{8000}\right) = 1 + \frac{\pi}{10} + \frac{\pi^2}{200} + \frac{\pi^3}{3000} \quad$ (*)	A1(cso) (2)
		8

Question number	Scheme	Marks
3.	(a) $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 3 \\ -4 \end{pmatrix}$ $\quad\overrightarrow{AC} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$	M1
	$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 3 & -4 \\ 2 & -2 & 3 \end{vmatrix} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ A1: One value correct, A1: All correct	M1 A1 A1 (4)
	(b) $\mathbf{r}.\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}.\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 3 - 4 + 8$ $\quad\mathbf{r}.\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 7$	M1 A1ft (2)
	(c) $\overrightarrow{AD}\,.\,\overrightarrow{AB} \times \overrightarrow{AC}$ (Attempt suitable triple scalar product)	M1
	$\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ or $\begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$ (if using AD)	B1
	Volume $= \frac{1}{6}\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}.\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = \frac{1}{6}(2 + 12 - 2) = 2$	M1 A1(cso) (4)
		10

Question number	Scheme	Marks	
4.	(a) $n = 1$: $\frac{d}{dx}\left(e^x \cos x\right) = e^x \cos x - e^x \sin x$ (Use of product rule)	M1	
	$\cos\left(x + \frac{\pi}{4}\right) = \cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}\left(\cos x - \sin x\right)$	M1	
	$\frac{d}{dx}\left(e^x \cos x\right) = 2^{\frac{1}{2}} e^x \cos\left(x + \frac{\pi}{4}\right)$ True for $n = 1$ (c.s.o. + comment)	A1	
	Suppose true for $n = k$.		
	$\left[\frac{d^{k+1}}{dx^{k+1}}\left(e^x \cos x\right)\right] = \frac{d}{dx}\left(2^{\frac{1}{2}k} e^x \cos\left(x + \frac{k\pi}{4}\right)\right)$	M1	
	$= 2^{\frac{1}{2}k}\left[e^x \cos\left(x + \frac{k\pi}{4}\right) - e^x \sin\left(x + \frac{k\pi}{4}\right)\right]$	A1	
	$= 2^{\frac{1}{2}k} e^x \sqrt{2} \cos\left(x + \frac{k\pi}{4} + \frac{\pi}{4}\right) = 2^{\frac{1}{2}(k+1)} e^x \cos\left(x + (k+1)\frac{\pi}{4}\right)$	M1 A1	
	$\therefore$ True for $n = k + 1$, so true (by induction) for all n. (≥ 1)	A1(cso)	(8)
	(b) $1 + \left(\sqrt{2}\cos\frac{\pi}{4}\right)x + \frac{1}{2}\left(2\cos\frac{\pi}{2}\right)x^2 + \frac{1}{6}\left(2\sqrt{2}\cos\frac{3\pi}{4}\right)x^3 + \frac{1}{24}\left(4\cos\pi\right)x^4$	M1	
	(1) (0) (–2) (–4)		
	$e^x \cos x = 1 + x - \frac{1}{3}x^3 - \frac{1}{6}x^4$ (or equiv. fractions)	A2(1,0)	(3)
			11

Question number	Scheme	Marks
5.	$\mathbf{MM}^{\mathrm{T}} = \begin{pmatrix} 1 & 4 & -1 \\ 3 & 0 & p \\ a & b & c \end{pmatrix}\begin{pmatrix} 1 & 3 & a \\ 4 & 0 & b \\ -1 & p & c \end{pmatrix} = \begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix}$	
	(a) $\begin{pmatrix} 1 & 4 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ 0 \\ p \end{pmatrix} = 0 \Rightarrow p = 3$	M1 A1 (2)
	(b) $\begin{pmatrix} 1 & 4 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix} = k \Rightarrow k = 18$ (ft on their p, if used)	M1 A1ft (2)
	(c) 2 equations: $a + 4b - c = 0$ $3a + 3c = 0$	M1
	a and b in terms of c (or equiv.): $a = -c$ $b = \frac{1}{2}c$ (ft on their p)	M1 A1ft
	Using $\begin{pmatrix} a \\ b \\ c \end{pmatrix} . \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 18$ $(a^2 + b^2 + c^2 = 18)$: $a = 2\sqrt{2}, b = -\sqrt{2}, c = -2\sqrt{2}$	M1 A2(1,0) (6)
	(d) $\lvert \det \mathbf{M} \rvert = \lvert (3\sqrt{2}) - 4(-12\sqrt{2}) - 1(-3\sqrt{2}) \rvert = 54\sqrt{2}$	M1 A1(cso) (2)
		12
	<u>Alternatives:</u>	
	(c) Require $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ parallel to $\begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}, = \begin{pmatrix} 12 \\ -6 \\ -12 \end{pmatrix}$	M1, M1 A1
	(Then as in main scheme, scaling to give a, b and c.)	M1 A2(1,0) (6)
	(d) $\det(\mathbf{MM}^{\mathrm{T}}) = 18^3$, $\det \mathbf{M} = \det \mathbf{M}^{\mathrm{T}}$, $\lvert \det \mathbf{M} \rvert = 18\sqrt{18}$ $(=54\sqrt{2})$	M1 A1 (2)

Question number	Scheme	Marks
6.	(*a*) det **A** = 0 $\quad (3-\lambda)^2 - 1 = 0$	M1
	$\lambda^2 - 6\lambda + 8 = 0 \quad (\lambda-2)(\lambda-4) = 0 \quad \lambda = 2,\ \lambda = 4$	A1
	$\lambda = 2: \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0, \quad x + y = 0,$ Eigenvector $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ (or equiv.)	M1 A1
	$\lambda = 4: \quad \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0, \quad -x + y = 0,$ Eigenvector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (or equiv.)	A1 (5)
	(*b*) $\mathbf{P} = k\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ M: eigenvectors as columns, $k = \frac{1}{\sqrt{2}}$	M1, A1
	$\left\{ \mathbf{P}^{-1} = \mathbf{P}^{\mathrm{T}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \right\}$	
	$\mathbf{D} = \mathbf{P}^{-1}\mathbf{AP} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix},\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$	M1, M1 A1 (5)
	(*c*) 1. Rotation of $\frac{\pi}{4}$ clockwise (about (0, 0)).	
	2. Stretch, × 4 parallel to *x*-axis, × 2 parallel to *y*-axis.	
	3. Rotation of $\frac{\pi}{4}$ anticlockwise (about (0, 0)).	
	1. and 3. both rotation, or both reflection.	M1
	Correct angles, opposite sense or correct lines (reflection).	A1
	Stretch.	B1
	All correct, including order.	A1 (4)
		14

Question number	Scheme	Marks
7.	(*a*) $\arg z = \frac{\pi}{4} \quad \Rightarrow \quad z = \lambda + \lambda \mathrm{i}$ (or putting *x* and *y* equal at some stage)	B1
	$w = \frac{(\lambda+1)+\lambda \mathrm{i}}{\lambda + (\lambda+1)\mathrm{i}}$, and attempt modulus of numerator or denominator.	M1
	(Could still be in terms of *x* and *y*)	
	$\lvert(\lambda+1)+\lambda \mathrm{i}\rvert = \lvert\lambda + (\lambda+1)\mathrm{i}\rvert = \sqrt{(\lambda+1)^2+\lambda^2}$, $\quad \therefore \lvert w\rvert = 1$ (*)	A1, A1cso (4)
	(*b*) $w = \frac{z+1}{z+\mathrm{i}} \quad \Rightarrow \quad zw + w\mathrm{i} = z+1 \quad \Rightarrow \quad z = \frac{1-w\mathrm{i}}{w-1}$	M1
	$\lvert z\rvert = 1 \quad \Rightarrow \quad \lvert 1 - w\mathrm{i}\rvert = \lvert w-1\rvert$	M1 A1
	For $w = a + \mathrm{i}b$, $\quad \lvert(1+b) - a\mathrm{i}\rvert = \lvert(a-1)+\mathrm{i}b\rvert$	M1
	$\sqrt{(1+b)^2+a^2} = \sqrt{(a-1)^2+b^2}$	M1
	$b = -a \qquad$ Image is (line) $y = -x$	A1 (6)
	(c) 	B1 B1 (2)
	(d) $z = \mathrm{i}$ marked (*P*) on *z*-plane sketch.	B1
	$z = \mathrm{i} \quad \Rightarrow \quad w = \frac{1+\mathrm{i}}{2\mathrm{i}} = \frac{\mathrm{i}-1}{-2} = \frac{1}{2} - \frac{1}{2}\mathrm{i}$ marked (*Q*) on *w*-plane sketch.	B1 (2)
		14