Question number	Scheme	Marks
1. (a)	$\cosh^2 x - \sinh^2 x = \frac{1}{4}(e^x + e^{-x})^2 - \frac{1}{4}(e^x - e^{-x})^2$	M1
	$= \frac{1}{4}(e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x})$	A1
	$= 1$ *	A1 (3)
(b)	$\frac{1}{\sinh x} - 2\frac{\cosh x}{\sinh x} = 2 \quad \therefore 1 - (e^x + e^{-x}) = e^x - e^{-x}$	M1
	$\therefore 2e^x = 1$	A1
	Make x the subject of the formula, $x = \ln(\frac{1}{2}) = -\ln 2$	M1, A1 (4)
2. (a)	$a = 2, \quad b = 1, \quad c = 16$	B1, B1, B1 (3)
(b)	$\int_{-0.5}^{1.5} \frac{1}{(2x+1)^2 + 16} dx$	M1
	$= \left[\frac{1}{8}\arctan\left(\frac{2x+1}{4}\right)\right]_{-0.5}^{1.5}$	M1 A1
	$= \frac{\pi}{32}$	B1 (4)

Question number	Scheme	Marks
3. (a)	As $4 = 9(1-e^2)$, $\therefore e^2 = \frac{5}{9}$	**M1, A1**
	Uses *ae* to obtain that the foci are at $(\pm\sqrt{5}, 0)$	**M1 A1** **(4)**
(b)	$PS + PS' = e(PM + PM')$ M1 for single statement e.g, PS = ePM	**M1**
	$= e \times \frac{2a}{e}$ M1 needs complete method	**M1**
	$= 2a = 6$	**A1** **(3)**
4.	$\frac{dx}{dt} = \sinh t - 1$ $\frac{dy}{dt} = \sinh t + 1$	**M1 A1**
	$\frac{d^2x}{dt^2} = \cosh t$ $\frac{d^2y}{dt^2} = \cosh t$	**M1 A1**
	Use $\rho = \left\lvert \frac{(\dot{x}^2 + \dot{y}^2)^{\frac{3}{2}}}{\dot{x}\ddot{y} - \dot{y}\ddot{x}} \right\rvert, = \frac{[(\sinh t - 1)^2 + (\sinh t + 1)^2]^{\frac{3}{2}}}{\sinh t \cosh t - \cosh t - \sinh t \cosh t - \cosh t}$	**M1, A1**
	$(=(-)\sqrt{2}\cosh^2 t)$	
	When t = ln3, cosh t = 5/3 (or sinh t = 4/3)	**M1A1**
	$\therefore \rho = (-)\sqrt{2} \times \frac{25}{9}$	**A1** **(9)**

Question number	Scheme	Marks
5. (a)	Using product rule $\frac{dy}{dx} = (n-1)\sinh^{n-2} x\cosh^2 x + \sinh^n x$	M1
	Using $\cosh^2 x = 1+\sinh^2 x$ in derived expression	M1
	to obtain $\frac{dy}{dx} = (n-1)\sinh^{n-2} x(1+\sinh^2 x) + \sinh^n x$	
	and $\frac{dy}{dx} = (n-1)\sinh^{n-2} x + n\sinh^n x$ *	A1
		(3)
(b)	$[\sinh^{n-1} x\cosh x]_0^{ar\sinh 1} = \int_0^{ar\sinh 1} (n-1)\sinh^{n-2} x dx + \int_0^{ar\sinh 1} n\sinh^n x dx$	
	So $\cosh(ar\sinh 1) = (n-1)I_{n-2} + nI_n$	M1
	If sinh α =1 then cosh α = $\sqrt{1+\sinh^2\alpha} = \sqrt{2}$	
	$\therefore nI_n = \sqrt{2} - (n-1)I_{n-2}$ *	A1
	OR	(2)
	$\int_0^{ar\sinh 1} \sinh^{n-1} x\sinh x dx = \left[\sinh^{n-1} x\cosh x\right]_0^{ar\sinh 1} - (n-1)\int_0^{ar\sinh 1} \cosh^2 x\sinh^{n-2} x dx$	
	and use $\cosh^2 x = 1+\sinh^2 x$	M1
	collect $I_n + (n-1)I_n$ *to obtain* $nI_n = \sqrt{2} - (n-1)I_{n-2}$ *	A1
		(2)
(c)	$I_0 = \text{ar}\sinh 1$	B1
	$2I_2 = \sqrt{2} - I_0$	M1
	$4I_4 = \sqrt{2} - 3I_2$ and use with previous results to obtain…	M1
	$= \frac{1}{8}(3\text{ar}\sinh 1 - \sqrt{2})$ = 0.154 (either answer acceptable)	A1
		(4)

Question number	Scheme	Marks
6. (a)	$\frac{ds}{d\psi} = 3a\sec^3\psi\tan\psi$	**B1**
	But $\frac{dy}{ds} = \sin\psi$, $\therefore \frac{dy}{d\psi} = 3a\sec^2\psi\tan^2\psi$	**M1**
	$\therefore y = a\tan^3\psi + c$, and as $y = 0$ at $\psi = 0$, then $c = 0$.	**M1**
	$\therefore y = a\tan^3\psi$ *	**A1** **(4)**
(b)	Also $\frac{dx}{ds} = \cos\psi$, $\therefore \frac{dx}{d\psi} = 3a\sec^2\psi\tan\psi$	**M1**
	$\therefore x = \frac{3}{2}a\tan^2\psi + c$, or $\therefore x = \frac{3}{2}a\sec^2\psi + k$,	**M1 A1**
	and as $x = 0$ at $\psi = 0$, then $c = 0$ (or $k = -\frac{3}{2}$) $\therefore x = \frac{3}{2}a\tan^2\psi$	**B1** **(4)**
(c)	Convincingly eliminate ψ, eg $\tan^6\psi = \frac{y^2}{a^2} = \frac{x^3}{(\frac{3}{2}a)^3}$, or convincingly substitute into the printed answer	**M1**
	Then obtain (or conclude) $ay^2 = \frac{8}{27}x^3$ *	**A1** **(2)**

Question number	Scheme	Marks
7. (a)	$\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta \qquad \frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$	**B1**
	$s = \int\sqrt{(9a^2(c^4s^2 + s^4c^2))}d\theta$	**M1**
	$= 3a\int\sqrt{c^2s^2}d\theta$	**M1**
	$= 3a\int\cos\theta\sin\theta d\theta$	**A1**
	Total length $= 4\times\frac{3a}{2}[\sin^2\theta]_0^{\frac{\pi}{2}}$	**M1**
	$=6a$	**M1 A1**
		(7)
(b)	$A = 2\pi\int a\sin^3\theta\times 3a\cos\theta\sin\theta d\theta$	**M1 A1**
	$= 6\pi a^2\int\sin^4\theta\cos\theta d\theta$	
	$= \frac{6\pi a^2}{5}\left[\sin^5\theta\right]_0^{\frac{\pi}{2}}\times 2$	**M1**
	$= \frac{12\pi a^2}{5}$	**M1 A1**
		(5)

Question number	Scheme	Marks
8. (a)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\frac{c}{t^2}}{c} = -\frac{1}{t^2}$	**M1 A1**
	The normal to the curve has gradient t^2.	**B1**
	The equation of the normal is $y - \frac{c}{t} = t^2(x - ct)$	**M1**
	The equation may be written $y = t^2 x + \frac{c}{t} - ct^3$ *	**A1** **(5)**
(b)	Let Q be the point $(cq, c/q)$	
	Then $\frac{c}{q} - \frac{c}{t} = cqt^2 - ct^3$ and so $\frac{c(t-q)}{qt} = ct^2(q - t)$	**M1 A1**
	Attempt to find q, e.g. $(q-t)(t^2 qt + 1) = 0$ or quadratic formula	**M1**
	$\therefore q = t$ *or* $\frac{-1}{t^3}$	**A1**
	So Q has coordinates $(-\frac{c}{t^3}, -ct^3)$	**A1** **(5)**
	Alternatives	
	Eliminate x or y between $xy = c^2$ *and* $y = t^2 x + \frac{c}{t} - ct^3$.	**M1**
	So $t^2 x^2 + (\frac{c}{t} - ct^3)x - c^2 = 0$ *.or* $y^2 = (\frac{c}{t} - ct^3)y + c^2 t^2 = 0$	**A1**
	Then solve using formula to obtain	**M1**
	$x = ct$ *or* $\frac{-c}{t^3}$ *or* $y = \frac{c}{t}$ *or* $-ct^3$	**A1**
	So Q has coordinates $(-\frac{c}{t^3}, -ct^3)$	**A1** **(5)**

Question number	Scheme	Marks
8. (c)	$X = \frac{c}{2}(t - \frac{1}{t^3}), \quad Y = \frac{c}{2}(\frac{1}{t} - t^3)$	**M1**
	$\therefore \frac{X}{Y} = \frac{t^4 - 1}{t^3} \times \frac{t}{1 - t^4} = -\frac{1}{t^2}$ *	**A1**
		(2)
(d)	$XY = \frac{-c^2}{4}(\frac{1}{t^2} - t^2)^2$	**M1**
	$\therefore XY = \frac{-c^2}{4}\left(\frac{-X}{Y} + \frac{Y}{X}\right)^2 \quad \rightarrow 4xy + c^2\left(\frac{y}{x} - \frac{x}{y}\right)^2 = 0$ *	**A1**
		(2)