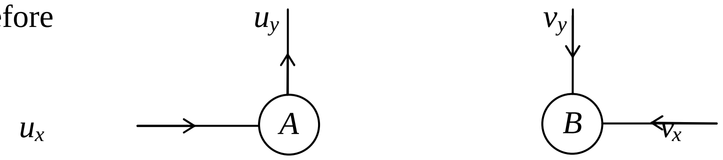
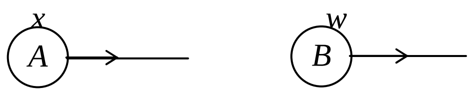
# EDEXCEL 6680 MECHANICS M4 JANUARY 2004 MARK SCHEME

| Question Number | Scheme | Marks |
|---|---|---|
| **1.** | N2L $\quad -2v = 3a$ | M1 |
| | $-2v = 3v\dfrac{\mathrm{d}v}{\mathrm{d}s}$ | A1 |
| | $s = -\dfrac{3}{2}v\,(+\,c)$ or $v = -\dfrac{2}{3}s\,(+\,c)$ cancelling $v$ and integrating | M1 |
| | $s = 0,\ v = 5 \Rightarrow c = \dfrac{15}{2}$ or $s = \left[-\dfrac{3}{2}v\right]_5^2$ | M1 |
| | Distance travelled is $4.5\,\mathrm{m}$ | A1 |
| | | **(5 marks)** |
| **2.** | (a) Before  | |
| | $A$: $\uparrow$ $u_y = 2.5\sin\alpha = 2.5 \times \frac{4}{5} = 2$ $\left(\mathrm{ms}^{-1}\right)$ either | M1 |
| | $\rightarrow$ $u_x = 2.5\cos\alpha = 2.5 \times \frac{3}{5} = 1.5$ $\left(\mathrm{ms}^{-1}\right)$ both | A1 |
| | $B$: $\downarrow$ $v_y = 1.3\sin\beta = 1.3 \times \frac{12}{13} = 1.2$ $\left(\mathrm{ms}^{-1}\right)$ either | M1 |
| | $\leftarrow$ $v_x = 1.3\cos\beta = 1.3 \times \frac{5}{13} = 0.5$ $\left(\mathrm{ms}^{-1}\right)$ both | A1 (4) |
| | (b) After  | |
| | LM $\quad 2x + w = 3 - 0.5\ (= 2.5)$ | M1 A1 ft |
| | NEL $\quad w - x = \frac{1}{2} \times 2\ (= 1)$ | M1 A1 ft |
| | Solving $\quad x = 0.5,\ y = 1.5$ $\quad$ M1 solving for either | M1 A1 |
| | Speed of $A$ is $\sqrt{\left(2^2 + 0.5^2\right)} = \sqrt{4.25} \approx 2.1$ $\left(\mathrm{m\,s}^{-1}\right)$ M1 either | M1 A1 |
| | Speed of $B$ is $\sqrt{\left(1.2^2 + 1.5^2\right)} = \sqrt{3.69} \approx 1.9$ $\left(\mathrm{m\,s}^{-1}\right)$ | A1 (9) |
| | | **(13 marks)** |
| | *Note*: *Not* 1 *d.p. loses maximum of one mark* | |

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|---|---|---|
| **3.** | (a) $AP = s - AD - DE$ | M1 |
| | $= s - L - 2L\sin\theta$ | A1 (2) |
| | (b) $V(\theta) = 2 \times 2mg \quad L\cos\theta \quad \ldots$ | B1 |
| | $= \quad - \quad \ldots \quad + mg(2L\cos\theta \quad AP)$ | M1 |
| | $= 4mgL\cos\theta + mg(2L\cos\theta \quad 2L\sin\theta)(+C)$ | M1 |
| | $+ \quad = 2mgL(3\cos\theta + \sin\theta)$ constant (*) cso | A1 (4) |
| | (c) $V`(\theta) = 2mgL\,(-3\sin\theta + \cos\theta)$ | M1 |
| | $= 0$ | M1 |
| | $\tan\theta = \frac{1}{3}$ | A1 |
| | $\theta \approx 18°$ awrt 18°, $0.32^{c}$ | A1 (4) |
| | (d) $V''(\theta) = 2mgL(-3\cos\theta \quad \sin\theta)$ | M1 A1 |
| | $\left(V''\left(\arctan\frac{1}{3}\right) = -2\sqrt{10} \quad mgL\right)$ | |
| | $V''(\theta) < 0$, for any acute $\theta$ | M1 |
| | Equilibrium is <u>unstable</u> ft any acute $\theta$ | A1 ft (4) |
| | | **(14 marks)** |

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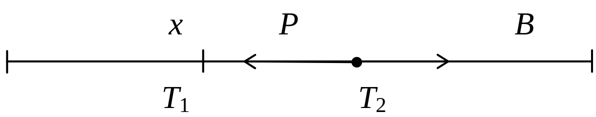
|---|---|---|---|---|
| **4.** | (a)  | | | |
| | HL $\quad T_1 = \dfrac{2mk^2L(0.5L + x)}{L}$ | either | M1 | |
| | HL $\quad T_2 = \dfrac{2mk^2L(0.5L - x)}{L}$ | both | A1 | |
| | N2L $\quad T_2 - T_1 - 2mk\dfrac{dx}{dt} = m\dfrac{d^2x}{dt^2}$ | | M1 A1, A1 | |
| | $4mk^2x - 2mk\dfrac{dx}{dt} = m\dfrac{d^2x}{dt^2}$ | | | |
| | $\dfrac{d^2x}{dt^2} + 2k\dfrac{dx}{dt} + 4k^2x = 0$ * | cso | A1 | (6) |
| | (b) $\quad m^2 + 2km + 4k^2 = 0$ | ae | M1 | |
| | $m = -k \pm \sqrt{3}k\,\mathrm{i}$ | | M1 | |
| | $x = e^{-kt}\left(A\cos\sqrt{3}kt + B\sin\sqrt{3}kt\right)$ | oe | A1 | |
| | $t = 0,\ x = \dfrac{L}{2} \Rightarrow A = \dfrac{L}{2}$ | | B1 | |
| | $\dot{x} = -k e^{-kt}\left(A\cos\sqrt{3}kt + B\sin\sqrt{3}kt\right)$ | | | |
| | $+ \sqrt{3}k e^{-kt}\left(-A\sin\sqrt{3}kt + B\cos\sqrt{3}kt\right)$ | | M1 | |
| | $t = 0,\ \dot{x} = 0 \Rightarrow 0 = -kA + \sqrt{3}kB$ | | M1 | |
| | $B = \dfrac{1}{\sqrt{3}}A = \dfrac{L}{2\sqrt{3}}$ | | A1 | |
| | $AP = 1.5L + \dfrac{L}{2\sqrt{3}}e^{-kt}\left(\sqrt{3}\cos\sqrt{3}kt + \sin\sqrt{3}kt\right)$ | oe | A1 | (8) |
| | | | | **(14 marks)** |
| | *Alternatives forms of the answer are given on the next page* | | | |

| Question Number | Scheme | Marks |
|---|---|---|
| **4.** | (b) *Alternative form of the General Solution* As before | M1 M1 |
| | $x = Ae^{-kt}\cos\left(\sqrt{3}kt - \varepsilon\right)$ | A1 |
| | $t = 0, x = \frac{L}{2} \Rightarrow \frac{L}{2} = A\cos(-\varepsilon) = A\cos\varepsilon)$ | B1 |
| | $\dot{x} = -kAe^{-kt}\cos\left(\sqrt{3}kt - \varepsilon\right) - \sqrt{3}kAe^{-kt}\sin\left(\sqrt{3}kt - \varepsilon\right)$ | M1 |
| | $t = 0, \dot{x} = 0 \Rightarrow 0 = -kA\cos\varepsilon - \sqrt{3}kA\sin(-\varepsilon)$ | M1 |
| | Leading to $\tan\varepsilon = \frac{1}{\sqrt{3}} \Rightarrow \varepsilon = \frac{\pi}{6}$ and $A = \frac{L}{\sqrt{3}}$ both | A1 |
| | $AP = 1.5L + \frac{L}{\sqrt{3}}e^{-kt}\cos\left(\sqrt{3}kt - \frac{\pi}{6}\right)$ | A1 (8) |
| | *Note: Another possible trig form is* $\sin\left(\sqrt{3}kt + \frac{\pi}{3}\right)$ | |
| **5.** | (a)Before After | |
| |  | |
| | $\rightarrow$ LM $600u = 800x$ | M1 A1 |
| | $\rightarrow$ NEL $x = eu$ | M1 A1 |
| | $e = 0.75$ | A1 (5) |
| | (b) Van N2L $-500 = 800a$ | M1 |
| | $0^2 = x^2 - 2\times 0.625\times 45$, $x^2 = 56.25$ $(x = 7.5)$ | M1, A1 |
| | Car N2L $-300 = 600a$ | M1 |
| | $0^2 = v^2 - 2\times 0.5\times 21$, $v^2 = 21$ | M1, A1 |
| | From (a) NEL $u = \frac{4}{3}\times 7.5 = 10$ | M1 |
| | $V^2 = 10^2 + 21$, $V = 11$ $\left(\text{ms}^{-1}\right)$ cao | M1, A1 (9) |
| | | **(14 marks)** |

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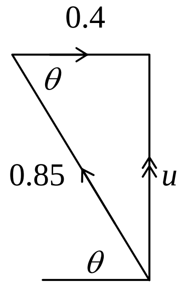
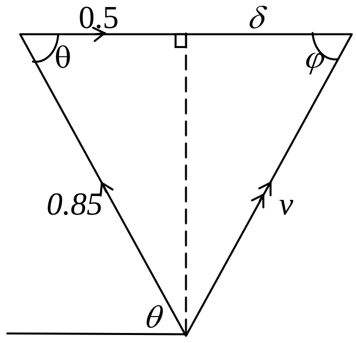
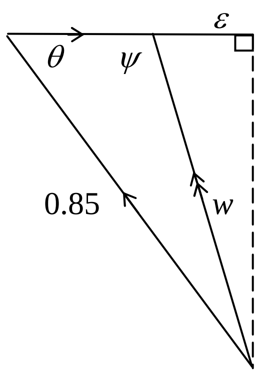
| **6.** |  (a) Vector ! or ← | M1 | |
| | $\cos\theta = \dfrac{0.4}{0.85}$ | M1 | |
| | $\theta \approx 61.9°$ awrt $62°$ | A1 | (3) |
| | (b) $u = \sqrt{(0.85^2 - 0.4^2)}$ or $u = 0.85\sin\theta$ | M1 | |
| | $t = \dfrac{60}{u} = \dfrac{60}{0.75} = 80$ (s) cao | M1 A1 | (3) |
| | (c) $\mathbf{v}_{N\,\mathrm{rel}\,W} = -0.4\mathbf{i}\ (+\ 0.75\mathbf{j})$ Allow for $\pm 0.4\mathbf{i}$ | M1 | |
| | $\mathbf{v}_N = \mathbf{v}_{N\,\mathrm{rel}\,W} + 0.5\mathbf{i} = 0.1\mathbf{i}\ (+0.75\mathbf{j})$ $0.1\mathbf{i}$ | A1 | |
| | $t = \dfrac{40}{0.75} = \dfrac{160}{3}$ | M1 | |
| | $\delta = 0.1 \times \dfrac{160}{3} = \dfrac{16}{3}$ awrt 5.3 | M1 A1 | (5) |
| | (d) As in (c) $\mathbf{v}_N = -0.2\mathbf{i} + 0.75\mathbf{j}$ $\pm 0.2\mathbf{i}$ | M1 | |
| | $t = \dfrac{20}{0.75} = \dfrac{80}{3}$ | M1 | |
| | $\delta = 0.2 \times \dfrac{80}{3} = \dfrac{16}{3}$ | M1 | |
| | Hence $N$ lands at $D$ cso | A1 | (4) |
| | | | **(15 marks)** |
| | *Notes*:<br>1. *In* (c) *and* (d), *the candidate can take components without using vectors. Mark as vector method.*<br>2. *After the first line in* (d), *the result is clear by proportion. Allow as long as some explanation given.*<br>3. $\cos\theta = \dfrac{8}{17} = 0.4705...$, $\sin\theta = \dfrac{15}{17} = 0.8823...$<br>4. *Alternatives to* (c) *and* (d), *using vector triangles are given on the next page.* | | |

| Question Number | Scheme | Marks |
|---|---|---|
| **6.** | *Alternatives to* (c) *and* (d) | |
| (c) | $v^2 = 0.5^2 + 0.85^2 - 2 \times 0.5 \times 0.85 \times \cos \theta$ | |
| | <br> | |
| | $= 0.5725$ ($v = \frac{\sqrt{229}}{20} \approx 82.4°$) | M1 |
| | $\frac{\sin\varphi}{0.85} = \frac{\sin\theta}{v}$ | M1 |
| | $\sin \varphi = \frac{15}{\sqrt{229}}$ $(\approx 0.9912;\ \varphi \approx 82.4°)$ | A1 |
| | $\frac{\delta}{40} = \cot\varphi;\quad \delta = 40 \times \frac{2}{5}\quad \frac{16}{3}$ awrt 5. 3 | M1 A1 (5) |
| (d) | $w^2 = 0.2^2 + 0.85^2 - 2 \times 0.2 \times 0.85 \times \cos \theta$ | |
| | $\approx\ = 0.6025\ \left(w = \frac{\sqrt{241}}{20}\quad 0.7762...\right)$ | M1 |
| | <br> | |
| | $\frac{\sin\psi}{0.85} = \frac{\sin\theta}{w}$ | |
| | $\sin \psi = \frac{15}{\sqrt{241}}$ $(\approx 0.9662;\ \psi \approx 104.9°$ | M1 |
| | $\psi = 75.1°$ gains M1 | |
| | $\frac{\varepsilon}{20} = \cot(180° - \psi) = \frac{4}{15}$ | |
| | $\varepsilon = \frac{16}{3} = \delta$ | M1 |
| | Hence $N$ lands at $D$ cso | A1 (4) |
| | *Note: Exact working is needed for final* A1 *but all previous marks in* (c) *and* (d) *may be gained by approximate working.* | |