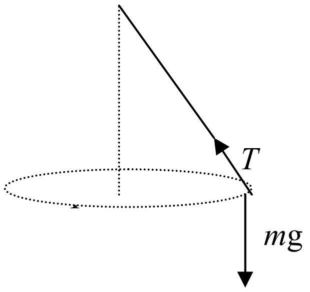
1.



(a) (↕) $T\cos 60° = mg \Rightarrow T = 2mg$ * — B1 **(1)**

(b) (↔) $T\sin 60° = mr\omega^2$ — M1A1
[Omission of m is M0]

Attempt at $r = L\sin 60°$ — M1

$(T\sin 60° = m\,L\sin 60°\,\omega^2)$

$\omega = \sqrt{\dfrac{2g}{L}}$ — A1 **(4)**

(c) Applying Hooke's Law: $2mg = \dfrac{\lambda}{(\frac{3}{5}L)}(L - \frac{2}{5}L)$; $\lambda = 3mg$ — M1;A1 **(2)**

[L in denominator is M0]

[7]

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2. (a) Integration of $-4e^{-2t}$ w.r.t. t to give $v = 2e^{-2t}$ (+c) — B1

Using initial conditions to find c (–1) or $v - 1 = [f(t)]_0^t$ — M1

$v = 2e^{-2t} - 1$ ms^{-1} — A1 **(3)**

(b) **Integrating** v w.r.t t; $x = -e^{-2t} - t$ (+c) — M1;A1√

Using $t = 0$, $x = 0$ **and** finding value for c ($c = 1$) — M1

Finding t when $v = 0$; $t = ½ \ln 2$ or equiv., 0.347 — M1;A1√

[both f.t. marks dependent on v of form $ae^{-2t} \pm b$]

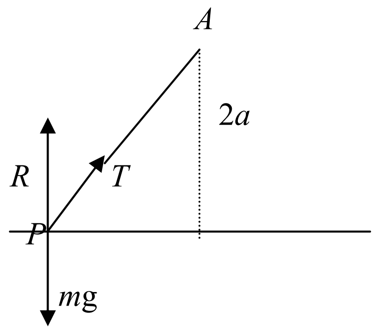
$x = ½(1 - \ln 2)$ m or 0.153 m(awrt) — A1 **(6)**

[9]

[For A1, exact form must be two termed answer]

3. (a) $F = \frac{k}{x^2}$ [k may be seen as Gm_1m_2 , for example]	M1
Equating F to mg at $\mathbf{x = R}$, [$mg = \frac{k}{R^2}$]	M1
Convincing completion [$k = mgR^2$] to give $F = \frac{mgR^2}{x^2}$ *	A1 **(3)**
[Note: r may be used instead of x throughout, then $r \to x$ at end.]	
(b) Equation of motion: $(m)a = (-)\frac{(m)gR^2}{x^2}$; $(m)v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{(m)gR^2}{x^2}$	M1;M1
Integrating: $\tfrac{1}{2}v^2 = \frac{gR^2}{x}$ (+ c) or equivalent	M1A1
[S.C: Allow A1√ if A0 earlier due to "+" only]	
Use of $v^2 = \frac{3gR}{2}$, $x = R$ to find c [$c = -\tfrac{1}{4}gR$] or use in def. int.	M1
[Using $x = 0$ is M0] [$v^2 = \frac{2gR^2}{x} - \frac{gR}{2}$]	
Substituting $x = 3R$ and finding V; $V = \sqrt{\frac{gR}{6}}$	M1;A1 **(7)**
[Using $x = 2R$ is M0]	
Alternative in (b)	**[10]**
Work/energy $(-)\int_R^a \frac{mgR^2}{x^2}\,\mathrm{d}x$; = $\tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$	M1;M1
Integrating: [$\frac{mgR^2}{x} - \frac{mgR^2}{R}$] = $\tfrac{1}{2}mv^2 - \tfrac{1}{2}m\frac{3gR}{2}$	M1A1M1
Final 2 marks as scheme	M1A1
[Conservation of energy scores 0]	

4.



(a) Length of string = $\frac{10}{3}a$ — B1

EPE = $\frac{\frac{1}{2}mg}{2a}(L - a)^2$ — M1

$= \frac{49}{36}mga$ — A1 **(3)**

(b) Energy equation: $\frac{1}{2}mv^2 + \frac{\frac{1}{2}mg}{2a}a^2 = (\frac{49}{36}mga)_C$ — M1A1☆

$v = \frac{2}{3}\sqrt{5ga}$ or equivalent — A1 **(3)**

(c) When string at angle θ to horizontal, length of string = $\frac{2a}{\sin\theta}$

$\Rightarrow$ Vert. Comp. of T, $T_V = T\sin\theta = \frac{mg}{2a}(\frac{2a}{\sin\theta} - a)\sin\theta$ — M1A1

$= \frac{mg}{2}(2 - \sin\theta)$

(↕) $R + T_V = mg$ and find R = …. — M1

$R = mg - \frac{mg}{2}(2 - \sin\theta) = \frac{mg}{2}\sin\theta$ — A1

$\Rightarrow R > 0$ (as $\sin\theta > 0$), so stays on table — A1 **(5)**

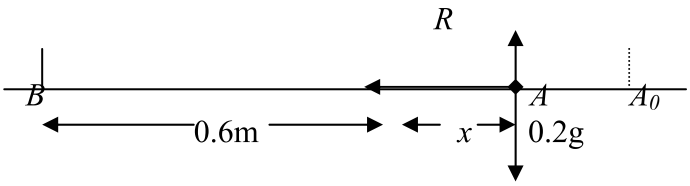
[Alternative final 3 marks: As θ increases so T_V decreases — M1

Initial T_V (string at β to hor.) = $\frac{7}{10}mg$ — A1

$\Rightarrow T_V \leq \frac{7}{10}mg < mg$, so stays on table A1]

[11]

5. (a)



$\lambda = 48$N

$BA_0 = 0.9$ m

Applying Hooke's Law correctly : e.g. $T = \dfrac{48x}{0.6}$	M1
Equation of motion: (–) T = 0.2 $\ddot{x}$	M1
Correct equation of motion: e.g. $-\dfrac{48x}{0.6} = 0.2\ \ddot{x}$	A1
Writing in form $\ddot{x} = -\ \omega^2\ x$, and stating motion is SHM	A1√
Period = $\dfrac{2\pi}{\omega} = \dfrac{2\pi}{20} = \dfrac{\pi}{10}$ * (no incorrect working seen)	A1 **(5)**
[If measure x from B or A, final 2 marks only available if equation of motion is reduced to $\ddot{X} = -\ \omega^2\ X$]	
(b) max $v = aw$ with values substituted; = 0.3 x 20 = 6 ms^{-1}	M1A1**(2)**
(c) Using $x = 0.3\cos 20t$ or $x = 0.3\sin 20T$	M1
Using $x = 0.15$ to give either cos $20t$ = ½ or sin $20T$ = ½	M1
Either $t = \dfrac{\pi}{60}, \dfrac{5\pi}{60}$ or $T = \dfrac{\pi}{120}$	A1
Complete method for time:	
$t_2 - t_1$, or $\dfrac{\pi}{10} - 2t_1$, or $2(\dfrac{\pi}{40} + T)$	M1
Time = $\dfrac{\pi}{15}$ s (must be in terms of π)	A1 **(5)**
	[12]

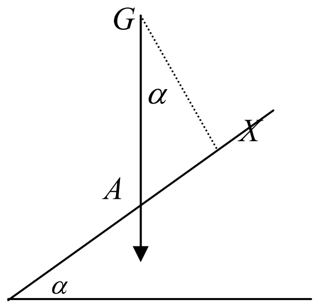
6. (a)

	Cylinder	Hemisphere	S	
Masses	$(\rho)\pi(2a)^2(\frac{3}{2}a)$	$(\rho)\frac{2}{3}\pi a^3$	$(\rho)(\frac{16}{3}\pi a^3)$	M1A1
	$[6\pi a^3]$ [18]	[2]	[16]	
Distance of CM from O	⅛ a	$\frac{3}{8}a$	$\bar{x}$	B1B1

Moments equation: $6\pi a^3(¾\, a) - \frac{2}{3}\pi a^3(\frac{3}{8}a) = \frac{16}{3}\pi a^3\,\bar{x}$ M1

$\bar{x} = \frac{51}{64}a$ A1 **(6)**

(b)



G above "A" seen or implied M1

or $mg\sin\alpha\,(GX) = mg\cos\alpha\,(AX)$

$\tan\alpha = \frac{AX}{XG} = \frac{2a}{\frac{3}{2}a - \bar{x}}$ M1

$[GX = \frac{3}{2}a - \frac{51}{64}a = \frac{45}{64}a$, $\tan\alpha = \frac{128}{45}]$ $\alpha = 70.6°$ A1 **(3)**

(c) Finding F and R : $R = mg\cos\beta$, $F = mg\sin\beta$ M1

Using $F = \mu R$ and finding $\tan\beta$ [= 0.8] M1

$\beta = 38.7°$ A1 **(3)**

[12]

7. (a) Energy: $\frac{1}{2} mv^2 - \frac{1}{2} mu^2 = mga \sin \theta$	M1	
$v^2 = \frac{3}{2} ga + 2ga \sin\theta$	A1	**(2)**
(b) Radial equation: $T - mg\sin\theta = m\frac{v^2}{a}$	M1A1	
$T = \frac{3mg}{2}(1 + 2\sin\theta)$ any form	A1☆	**(3)**
(c) Setting $T = 0$ and solving trig. equation; $(\sin\theta = -\frac{1}{2}) \Rightarrow \theta = 210°$ *	M1;A1	**(2)**
(d) Setting **v = 0** in (a) and solving for θ	M1	
$\sin\theta = -\frac{3}{4}$ so not complete circle	A1	**(2)**
OR Substituting $\theta = 270°$ in (a); $v^2 < 0$ so not possible to complete		
(e) No change in PE $\Rightarrow$ no change in KE (Cof E) so $v = u$	B1	**(1)**
(f) When string becomes slack, $V^2 = \frac{1}{2} ga$ $[\sin\theta = -\frac{1}{2}$ in (a)]	B1☆	
Using fact that horizontal component of velocity is unchanged	M1	
$\sqrt{\frac{ga}{2}} \cos 60^\circ = \sqrt{\frac{3ga}{2}} \cos\phi$		
$\cos\phi = \sqrt{\frac{1}{12}} \Rightarrow \phi = 73.2°$	M1A1	**(4)**
		[14]