Paper Reference(s)

6672

Edexcel GCE

Pure Mathematics P2

Advanced/Advanced Subsidiary

Friday 13 June 2003 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P2), the paper reference (6672), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. (*a*) Simplify $\dfrac{x^2 + 4x + 3}{x^2 + x}$.

(2)

(*b*) Find the value of x for which $\log_2 (x^2 + 4x + 3) - \log_2 (x^2 + x) = 4$.

(4)

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2. The functions f and g are defined by

$$\mathrm{f}\colon x \mapsto x^2 - 2x + 3,\ x \in \mathbb{R},\ \ 0 \le x \le 4,$$

$$\mathrm{g}\colon x \mapsto \lambda x + 1, \text{ where } \lambda \text{ is a constant, } x \in \mathbb{R}.$$

(*a*) Find the range of f.

(3)

(*b*) Given that $\mathrm{gf}(2) = 16$, find the value of λ.

(3)

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3. The expansion of $(2 - px)^6$ in ascending powers of x, as far as the term in x^2, is

$$64 + Ax + 135x^2.$$

Given that $p > 0$, find the value of p and the value of A.

(7)

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4. **Figure 1**

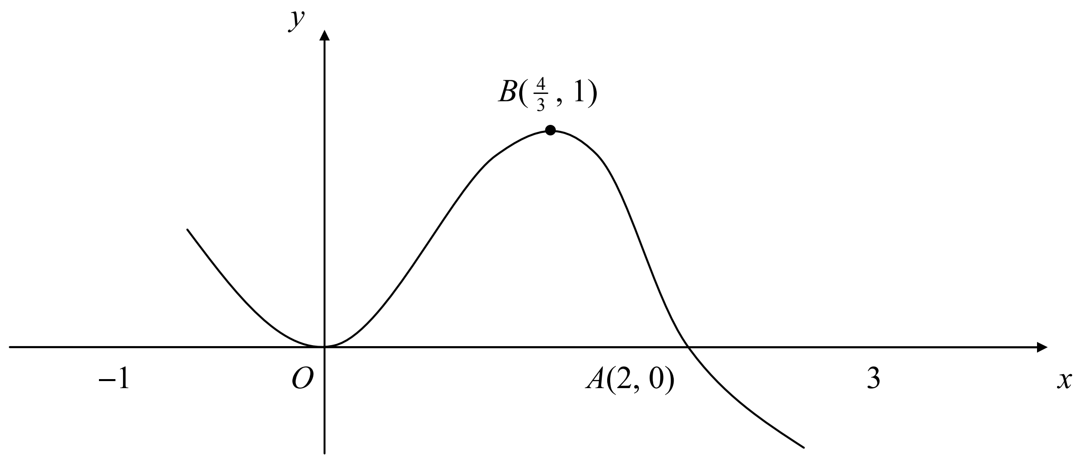


Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, $-1 \leq x \leq 3$. The curve touches the x-axis at the origin O, crosses the x-axis at the point $A(2, 0)$ and has a maximum at the point $B(\frac{4}{3}, 1)$.

In separate diagrams, show a sketch of the curve with equation

(*a*) $y = \mathrm{f}(x + 1)$,

(3)

(*b*) $y = |\mathrm{f}(x)|$,

(3)

(*c*) $y = \mathrm{f}(|x|)$,

(4)

marking on each sketch the coordinates of points at which the curve

(i) has a turning point,

(ii) meets the x-axis.

5. (*a*) Sketch, on the same set of axes, the graphs of

$$y = 2 - e^{-x} \text{ and } y = \sqrt{x}.$$

(3)

[It is not necessary to find the coordinates of any points of intersection with the axes.]

Given that $f(x) = e^{-x} + \sqrt{x} - 2,\ x \geq 0$,

(*b*) explain how your graphs show that the equation $f(x) = 0$ has only one solution,

(1)

(*c*) show that the solution of $f(x) = 0$ lies between $x = 3$ and $x = 4$.

(2)

The iterative formula $x_{n+1} = (2 - e^{-x_n})^2$ is used to solve the equation $f(x) = 0$.

(*d*) Taking $x_0 = 4$, write down the values of x_1, x_2, x_3 and x_4, and hence find an approximation to the solution of $f(x) = 0$, giving your answer to 3 decimal places.

(4)

6.

Figure 2

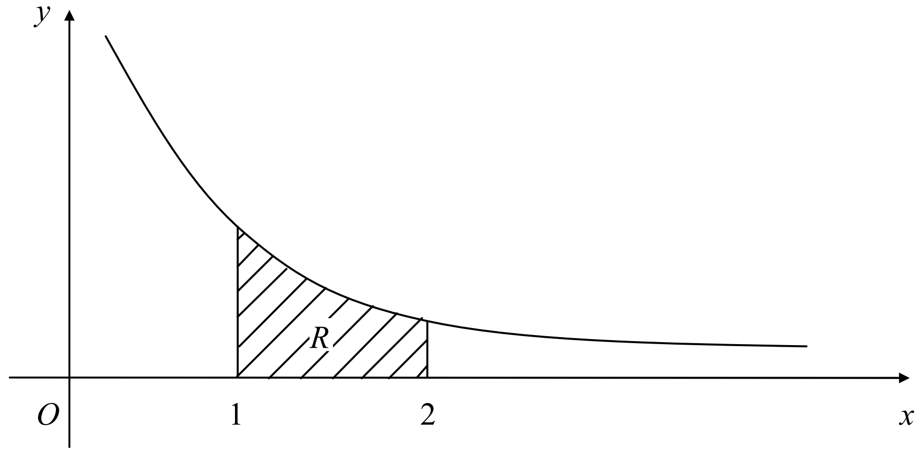


Figure 2 shows part of the curve with equation $y = 1 + \dfrac{c}{x}$, where c is a positive constant.

The point P with x-coordinate p lies on the curve. Given that the gradient of the curve at P is -4,

(a) show that $c = 4p^2$.

(2)

Given also that the y-coordinate of P is 5,

(b) prove that $c = 4$.

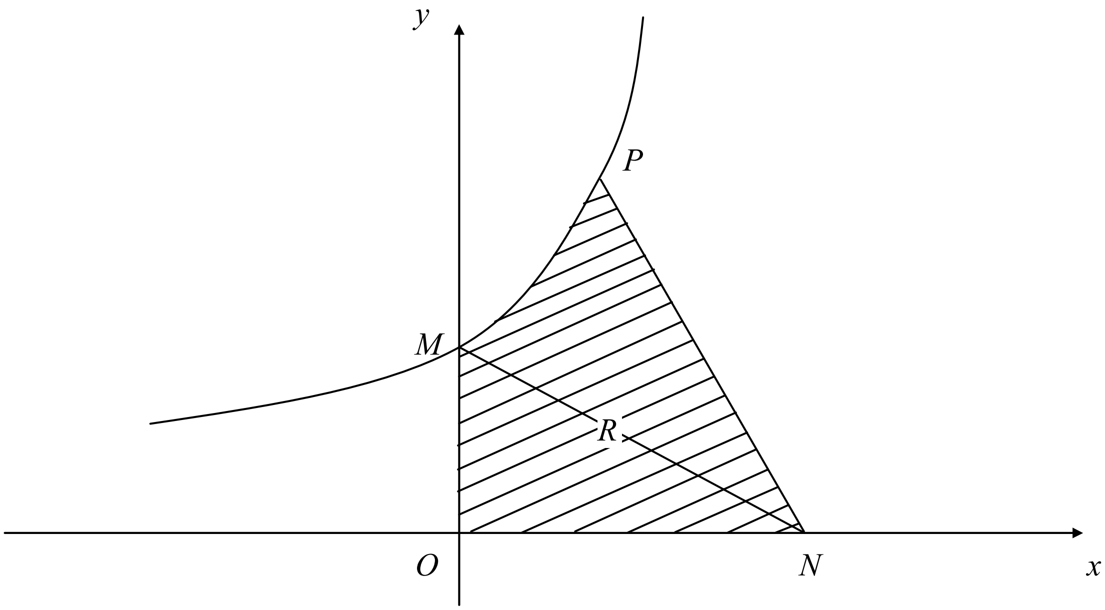
(2)

The region R is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$, as shown in Fig. 2. The region R is rotated through 360° about the x-axis.

(c) Show that the volume of the solid generated can be written in the form $\pi(k + q \ln 2)$, where k and q are constants to be found.

(7)

7. **Figure 3**



The curve C with equation $y = 2e^x + 5$ meets the y-axis at the point M, as shown in Fig. 3.

(a) Find the equation of the normal to C at M in the form $ax + by = c$, where a, b and c are integers.

(4)

This normal to C at M crosses the x-axis at the point $N(n, 0)$.

(b) Show that $n = 14$.

(1)

The point $P(\ln 4, 13)$ lies on C. The finite region R is bounded by C, the axes and the line PN, as shown in Fig. 3.

(c) Find the area of R, giving your answers in the form $p + q \ln 2$, where p and q are integers to be found.

(7)

8. (i) Given that $\cos(x + 30)° = 3\cos(x - 30)°$, prove that $\tan x° = -\dfrac{\sqrt{3}}{2}$.

(5)

(ii) (*a*) Prove that $\dfrac{1 - \cos 2\theta}{\sin 2\theta} \equiv \tan\theta$.

(3)

(*b*) Verify that $\theta = 180°$ is a solution of the equation $\sin 2\theta = 2 - 2\cos 2\theta$.

(1)

(*c*) Using the result in part (*a*), or otherwise, find the other two solutions, $0 < \theta < 360°$, of the equation using $\sin 2\theta = 2 - 2\cos 2\theta$.

(4)

END