Paper Reference(s)

6675

Edexcel GCE

Pure Mathematics P5

Advanced/Advanced Subsidiary

Friday 13 June 2003 – Morning

Time: 1 hour 30 minutes

<u>**Materials required for examination**</u>	<u>**Items included with question papers**</u>
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P5), the paper reference (6675), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. Find the values of x for which

$$4 \cosh x + \sinh x = 8,$$

giving your answer as natural logarithms.

(6)

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2. (a) Prove that the derivative of artanh x, $-1 < x < 1$, is $\dfrac{1}{1-x^2}$.

(3)

(b) Find $\int \operatorname{artanh} x \, \mathrm{d}x$.

(4)

3.

Figure 1

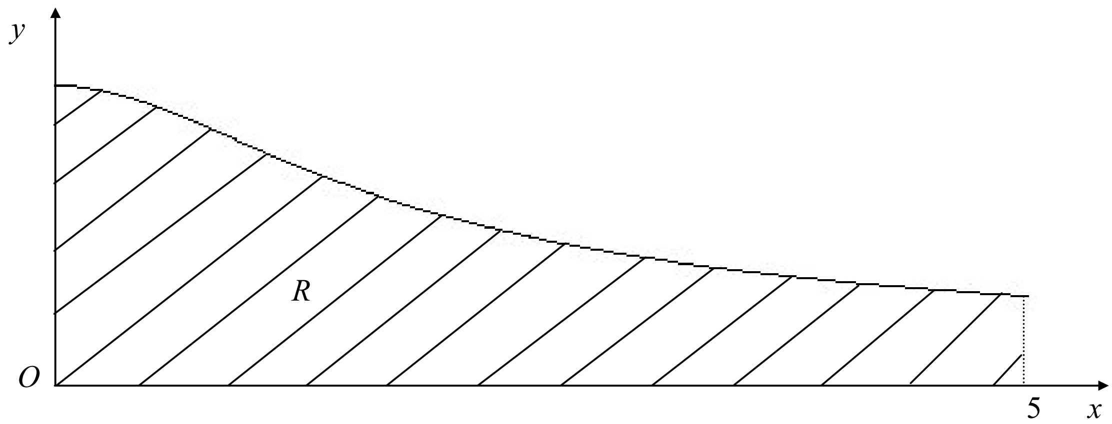

Figure 1 shows the cross-section R of an artificial ski slope. The slope is modelled by the curve with equation

$$y = \frac{10}{\sqrt{(4x^2+9)}}, \qquad 0 \leq x \leq 5.$$

Given that 1 unit on each axis represents 10 metres, use integration to calculate the area R. Show your method clearly and give your answer to 2 significant figures.

(7)

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4. The radius of curvature of a curve C, at any point P on C, is $2\cos\psi$, where ψ is the angle between the tangent to C at P and the positive x-axis, and $0 \leq \psi < \dfrac{\pi}{2}$.

Taking $s = 0$ at $\psi = \dfrac{\pi}{6}$,

(a) find an intrinsic equation for C.

(4)

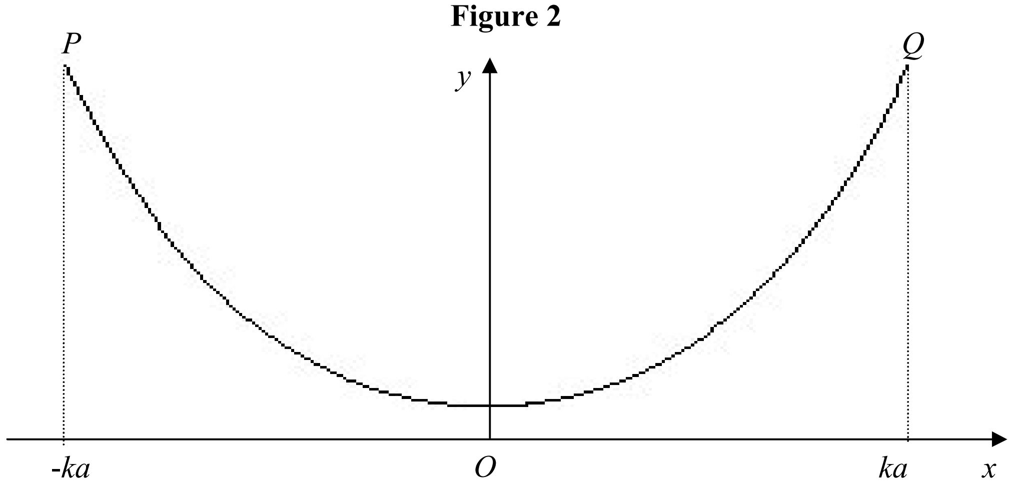
Given also that $s = 0$ at $y = 0$,

(b) find y in terms of s.

(4)

5.

Figure 2



A rope is hung from points P and Q on the same horizontal level, as shown in Fig. 2. The curve formed by the rope is modelled by the equation

$$y = a\cosh\left(\frac{x}{a}\right), \qquad -ka \leq x \leq ka,$$

where a and k are positive constants.

(a) Prove that the length of the rope is $2a \sinh k$.

(5)

Given that the length of the rope is $8a$,

(b) find the coordinates of Q, leaving your answer in terms of natural logarithms and surds, where appropriate.

(4)

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6. The curve C has equation

$$y = \operatorname{arcsec} e^x, \qquad x > 0, \qquad 0 < y < \tfrac{1}{2}\pi .$$

(a) Prove that $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{(e^{2x} - 1)}}$.

(5)

(b) Sketch the graph of C.

(2)

The point A on C has x-coordinate ln 2. The tangent to C at A intersects the y-axis at the point B.

(c) Find the exact value of the y-coordinate of B.

(4)

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7. $$I_n = \int_0^1 x^n e^x \, dx \quad \text{and } J_n = \int_0^1 x^n e^{-x} \, dx, \quad n \geq 0.$$

(a) Show that, for $n \geq 1$,

$$I_n = e - nI_{n-1}.$$

(2)

(b) Find a similar reduction formula for J_n .

(3)

(c) Show that $J_2 = 2 - \dfrac{5}{e}$.

(3)

(d) Show that $\displaystyle\int_0^1 x^n \cosh x \, dx = \tfrac{1}{2}(I_n + J_n)$.

(1)

(e) Hence, or otherwise, evaluate $\displaystyle\int_0^1 x^2 \cosh x \, dx$, giving your answer in terms of e.

(4)

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8. The hyperbola C has equation $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$.

(*a*) Show that an equation of the normal to C at the point P $(a \sec t, b \tan t)$ is

$$ax \sin t + by = (a^2 + b^2) \tan t.$$

(6)

The normal to C at P cuts the x-axis at the point A and S is a focus of C. Given that the eccentricity of C is $\frac{3}{2}$, and that $OA = 3OS$, where O is the origin,

(*b*) determine the possible values of t, for $0 \le t < 2\pi$.

(8)

END