Question Number	Scheme	Marks
1.	$4\left(\frac{e^x + e^{-x}}{2}\right) + \frac{e^x - e^{-x}}{2} = 8$	M1
	$5e^{2x} - 16e^x + 3 = 0$	M1 A1
	$(5e^x - 1)(e^x - 3) = 0$	A1
	$e^x = \frac{1}{5}, 3$	
	$x = \ln(\frac{1}{5}), \ln 3$ accept $-\ln 5$	M1 A1 (6)
		(6 marks)
2. (*a*)	$y = \text{artanh}\, x$	
	$\tanh y = x$	
	$\text{sech}^2 y \frac{dy}{dx} = 1$	M1 A1
	$\frac{dy}{dx} = \frac{1}{1 - \tanh^2 y} = \frac{1}{1 - x^2}$ (*)	A1 (3)
	cso	
(*b*)	$\int 1.\text{artanh}\, x\, dx = x\, \text{artanh}\, x - \int \frac{x}{1 - x^2}\, dx$	M1 A1
	$= x\, \text{artanh}\, x + \frac{1}{2} \ln(1 - x^2)\ (+ c)$	M1 A1 (4)
		(7 marks)
Alt.	$\frac{x}{1 - x^2} \equiv \frac{1}{2}\left[\frac{1}{1 - x} - \frac{1}{1 + x}\right]$	
	$\int \frac{x}{1 - x^2}\, dx = -\frac{1}{2}\ln(1 - x) - \frac{1}{2}\ln(1 + x)$	
	This is acceptable (with the rest correct) for final M1 A1	

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Question Number	Scheme	Marks
3.	$\int \frac{10}{\sqrt{4x^2+9}}\,dx = \frac{1}{2}\int \frac{10}{\sqrt{x^2+\frac{9}{4}}}\,dx$	M1
	$= \frac{10}{2}\text{arsinh}\left(\frac{2x}{3}\right) \quad \left(= 5\ln\left[\frac{2x}{3}+\sqrt{\frac{4x^2}{9}+1}\right]\right)$	M1 A1
	$[\ ]_0^5 = 5\text{ arsinh}\frac{10}{3} \quad \left(= 5\ln\left(\frac{10}{3}+\sqrt{\frac{109}{9}}\right) \approx 9.594\right)$ ft on 5	M1 A1 ft
	Area = 9.594 × 100 = 960 (m^2)	M1 A1
		(7 marks)
	Using a substitution	
	(i) $2x = 3\sinh\theta$; $2\,dx = 3\cosh\theta\,d\theta$	
	$\int \frac{10}{\sqrt{4x^2+9}}\,dx = \int \frac{10}{3\cosh\theta}\times\frac{3}{2}\cosh\theta\,d\theta$ complete subs.	M1
	$= 5\int d\theta = 5\text{ arsinh}\frac{2x}{3}$	M1 A1
	then as before,	
	or changing limits to 0 and arsinh $\frac{10}{3}$ (or $\ln\left(\frac{10}{3}+\sqrt{\frac{109}{9}}\right)$) can gain this A1	
	(ii) $2x = 3\tan\theta$; $2\,dx = 3\sec^2\theta\,d\theta$	
	$\int \frac{10}{\sqrt{4x^2+9}}\,dx = \int \frac{10}{\sqrt{9\tan^2\theta+9}}\times\frac{3}{2}\sec^2\theta\,d\theta$	M1
	$= 5\int \sec\theta\,d\theta = 5\ln(\sec\theta+\tan\theta)$	M1
	Limits are 0 and arctan $\frac{10}{3}$	A1
	$[\ ]_0^{\arctan\frac{10}{3}} = 5\ln\left(\sqrt{\frac{100}{9}+1}+\frac{10}{3}\right)$ etc	M1 A1ft

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Question Number	Scheme	Marks
4. (*a*)	$\rho = \dfrac{ds}{d\psi} = 2\cos\psi$	M1
	$s = 2\sin\psi \ (+c)$	A1
	$\left(0, \frac{\pi}{6}\right) \Rightarrow 0 = 1 + c$	M1
	$s = 2\sin\psi - 1$	A1 (4)
(*b*)	$\dfrac{dy}{ds} = \sin\psi = \frac{1}{2}s + \frac{1}{2}$ ft their $\sin\psi$ only	M1 A1ft
	$y = \frac{1}{4}s^2 + \frac{1}{2}s \ (+c)$	M1
	initial conditions $\Rightarrow c = 0$	
	$y = \frac{1}{4}s^2 + \frac{1}{2}s$	A1 (4)
		(8 marks)
5. (*a*)	$\dfrac{dy}{dx} = \sinh\dfrac{x}{a}$	B1
	$\displaystyle\int\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int\sqrt{1+\sinh^2\left(\frac{x}{a}\right)}\,dx = \int\cosh\frac{x}{a}\,dx = \sinh\frac{x}{a}$	M1, A1
	$\text{Length} = 2\left[a\sinh\dfrac{x}{a}\right]_0^{ka} = 2a\sinh k \quad (*)$	M1 A1 (5)
(*b*)	$2a\sinh k = 8a$	
	$\sinh k = 4$	B1
	$x = ka = a\,\text{arsinh}\,4 = a\ln(4 + \sqrt{17})$	B1
	$y = a\cosh\dfrac{ka}{a} = a\sqrt{(1+\sinh^2 k)} = a\sqrt{17}$	M1 A1 (4)
		(9 marks)

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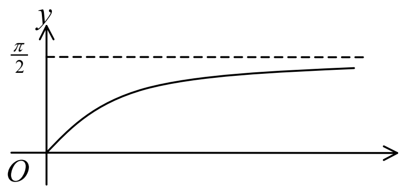
Question Number	Scheme	Marks
6. (*a*)	$\sec y = e^x$	B1
	$\sec y \tan y \dfrac{dy}{dx} = e^x \quad (= \sec y)$	M1 A1
	$\dfrac{dy}{dx} = \dfrac{e^x}{\sec y \tan y} = \dfrac{1}{\sqrt{\sec^2 y - 1}} = \dfrac{1}{\sqrt{e^{2x} - 1}}$ (*) cso	M1 A1 (5)
	 Shape, curve → (0, 0)	B1
	Asymptote, $(y =)\ \dfrac{\pi}{2}$	B1 (2)
(*c*)	$(x = \ln 2) \quad y = \text{arcsec } 2 = \dfrac{\pi}{3}$	B1
	$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{4-1}} = \dfrac{1}{\sqrt{3}}$	B1
	tangent is $\quad y - \dfrac{\pi}{3} = \dfrac{1}{\sqrt{3}}(x - \ln 2)$	M1
	$x = 0, \quad y = \dfrac{\pi}{3} - \dfrac{1}{\sqrt{3}} \ln 2$ exact answer only	A1 (4)
		(11 marks)
Alt. to (*a*)	$\cos y = e^{-x}$	B1
	$-\sin y \dfrac{dy}{dx} = \dfrac{e^{-x}}{\sqrt{1 - \cos^2 y}} = \dfrac{e^{-x}}{\sqrt{1 - e^{-2x}}} = \dfrac{1}{\sqrt{e^{2x} - 1}}$ (*) cso	M1 A1
		M1 A1 (5)

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Question Number	Scheme	Marks
7. (*a*)	$I_n = \left[x^n e^x\right]_0^1 - n\int_0^1 x^{n-1}e^x \, dx = e - nI_{n-1}$ (*) cso	M1 A1 (2)
(*b*)	$J_n = \left[-x^n e^{-x}\right]_0^1 + n\int_0^1 x^{n-1}e^{-x} \, dx$	M1 A1
	$= -e^{-1} + nJ_{n-1}$	A1 (3)
(*c*)	$J_2 = -e^{-1} + 2J_1$	
	$J_1 = -e^{-1} + J_0$ J_2 <u>and</u> J_1	M1
	$= -e^{-1} + \int_0^1 e^{-x} \, dx$	
	$= -e^{-1} + (1 - e^{-1}) \quad (= 1 - 2e^{-1})$	A1
	$J_2 = -e^{-1} + 2(1 - 2e^{-1}) = 2 - \dfrac{5}{e}$ (*)	A1 (3)
(*d*)	$\int_0^1 x^n \cosh x \, dx = \int_0^1 x^n \left(\dfrac{e^x + e^{-x}}{2}\right) dx = \frac{1}{2}(I_n + J_n)$ (*)	B1 (1)
(*e*)	$I_2 = e - 2I_1 = e - 2(e - I_0) = 2I_0 - e$	
	$= 2\int_0^1 e^x \, dx - e = 2[e - 1] - e \quad (= e - 2)$	M1 A1
	$\frac{1}{2}(I_2 + J_2) = \frac{1}{2}(e - 2 + 2 - \dfrac{5}{e}) = \frac{1}{2}(e - \dfrac{5}{e})$	M1 A1 (4)
		(13 marks)
Alt. to (*e*)	$\int_0^1 x^2 \cosh x \, dx = \left[x^2 \sinh x\right]_0^1 - \int_0^1 2x \sinh x \, dx$	
	$= \left[x^2 \sinh x\right]_0^1 - \{\left[2x \cosh x\right]_0^1 - \int_0^1 2\cosh x \, dx\}$	
	$= \left[x^2 \sinh x - 2x\cosh x + 2\sinh x\right]_0^1$	M1 A1
	$= \sinh 1 - 2\cosh 1 + 2\sinh 1$	M1
	$= 3\sinh 1 - 2\cosh 1$	
	$= 3\left(\dfrac{e - e^{-1}}{2}\right) - 2\left(\dfrac{e + e^{-1}}{2}\right) = \frac{1}{2}(e - \dfrac{5}{e})$	A1 (4)

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Question Number	Scheme	Marks
8. (*a*)	$\frac{dx}{dt} = a\sec t\tan t, \quad \frac{dy}{dt} = b\sec^2 t$	M1 A1
	$\frac{dy}{dx} = \frac{b\sec^2 t}{a\sec t\tan t} \quad \left(= \frac{b}{a\sin t}\right)$	M1 A1
	gradient of normal is $-\frac{a\sin t}{b}$	
	$y - b\tan t = -\frac{a\sin t}{b}(x - a\sec t)$	M1
	$ax\sin t + by = (a^2 + b^2)\tan t$ (*) cso	A1 (6)
(*b*)	$y = 0 \Rightarrow \quad x = \frac{(a^2+b^2)\tan t}{a\sin t} \quad \left(= \frac{a^2+b^2}{a\cos t}\right)$	B1
	$b^2 = a^2(e^2 - 1) \quad \Rightarrow \quad b^2 = \frac{5a^2}{4}$	M1
	$OS = ae$ and $OA = 3AS$	M1
	$a^2 + \frac{5a^2}{4} = 3a^2 \times \frac{3}{2} \times \cos t$	
	$\cos t = \frac{1}{2}$	M1 A1
	$t = \frac{\pi}{3}, \frac{5\pi}{3}$	A1
	By symmetry or (as $OA = \left\lvert\frac{a^2+b^2}{a\cos t}\right\rvert$) $-\frac{a^2+b^2}{a\cos t} = 3ae$	
	$t = \frac{2\pi}{3}, \frac{4\pi}{3}$	M1 A1 (8)
		(14 marks)
Alt. to (*a*)	$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$	M1 A1
	$\frac{dy}{dx} = \frac{2b^2x}{2a^2y} = \frac{b^2 a\sec t}{a^2 b\tan t} \ldots$	M1 A1
	then as before	

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