Paper Reference(s)

6674

Edexcel GCE

Pure Mathematics P4

Advanced/Advanced Subsidiary

Monday 23 June 2003 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P4), the paper reference (6674), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. (*a*) Express as a simplified single fraction $\frac{1}{(r-1)^2} - \frac{1}{r^2}$.

(2)

(*b*) Hence prove, by the method of differences, that

$$\sum_{r=2}^{n} \frac{2r-1}{r^2(r-1)^2} = 1 - \frac{1}{n^2}.$$

(3)

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2. Solve the inequality $\frac{1}{2x+1} > \frac{x}{3x-2}$.

(6)

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3. (*a*) By factorisation, show that two of the roots of the equation $x^3 - 27 = 0$ satisfy the quadratic equation $x^2 + 3x + 9 = 0$.

(2)

(*b*) Hence, or otherwise, find the three cube roots of 27, giving your answers in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{R}$.

(3)

(*c*) Show these roots on an Argand diagram.

(2)

4.

$$\mathrm{f}(x) = 3^x - x - 6.$$

(*a*) Show that $\mathrm{f}(x) = 0$ has a root α between $x = 1$ and $x = 2$.

(2)

(*b*) Starting with the interval (1, 2), use interval bisection three times to find an interval of width 0.125 which contains α.

(2)

(*c*) Taking 2 as a first approximation to α, apply the Newton-Raphson procedure once to $\mathrm{f}(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.

(4)

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5. $$z = \frac{a + 3\text{i}}{2 + a\text{i}}, \qquad a \in \mathbb{R}.$$

(a) Given that $a = 4$, find $|z|$.

(3)

(b) Show that there is only one value of a for which $\arg z = \frac{\pi}{4}$, and find this value.

(6)

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6. (a) Using the substitution $t = x^2$, or otherwise, find

$$\int x^3 \text{e}^{-x^2} \, \text{d}x .$$

(6)

(b) Find the general solution of the differential equation

$$x\frac{\text{d}y}{\text{d}x} + 3y = x\text{e}^{-x^2}, \quad x > 0.$$

(4)

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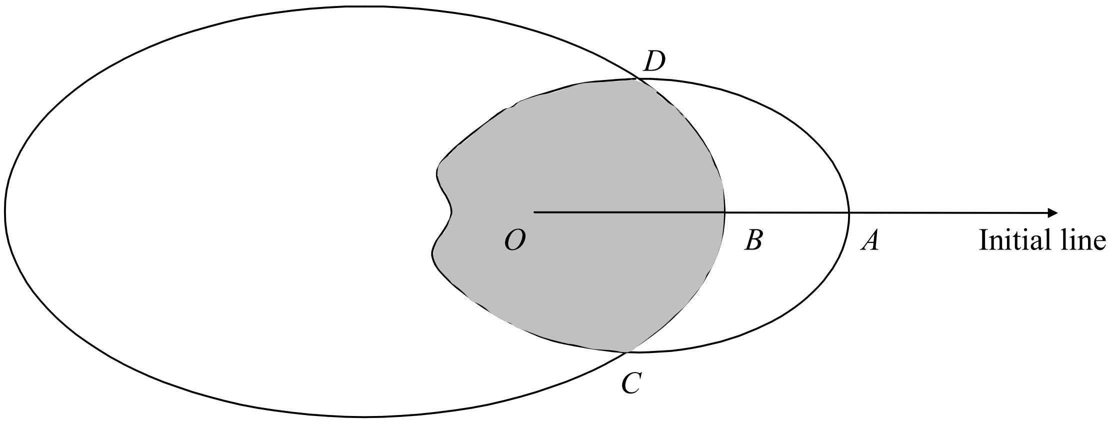
7. **Figure 1**



A logo is designed which consists of two overlapping closed curves.

The polar equations of these curves are

$$r = a(3 + 2\cos\theta) \quad \text{and}$$

$$r = a(5 - 2\cos\theta), \quad 0 \le \theta < 2\pi.$$

Figure 1 is a sketch (not to scale) of these two curves.

(*a*) Write down the polar corrdinates of the points *A* and *B* where the curves meet the initial line. **(2)**

(*b*) Find the polar coordinates of the points *C* and *D* where the two curves meet. **(4)**

(*c*) Show that the area of the overlapping region, which is shaded in the figure, is

$$\frac{a^2}{3}(49\pi - 48\sqrt{3}).$$

(8)

8. $$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = 4e^{3t}, \quad t \geq 0.$$

(*a*) Show that Kt^2e^{3t} is a particular integral of the differential equation, where K is a constant to be found.

(4)

(*b*) Find the general solution of the differential equation.

(3)

Given that a particular solution satisfies $y = 3$ and $\frac{dy}{dt} = 1$ when $t = 0$,

(*c*) find this solution.

(4)

Another particular solution which satisfies $y = 1$ and $\frac{dy}{dt} = 0$ when $t = 0$, has equation

$$y = (1 - 3t + 2t^2)e^{3t}.$$

(*d*) For this particular solution draw a sketch graph of y against t, showing where the graph crosses the t-axis. Determine also the coordinates of the minimum of the point on the sketch graph.

(5)

END