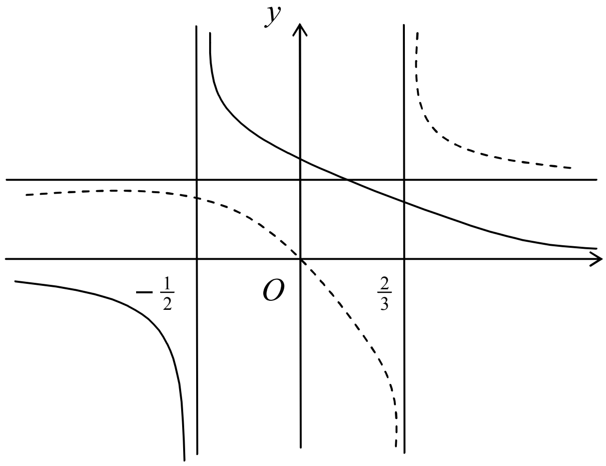
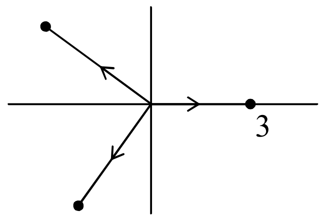
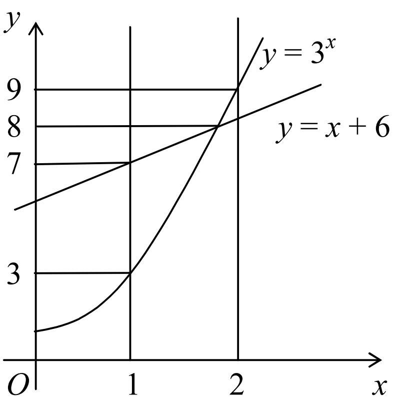
Question Number	Scheme	Marks
1. (*a*)	$\dfrac{r^2-(r-1)^2}{r^2(r-1)^2}=\dfrac{2r-1}{r^2(r-1)^2}$	M1, A1 (2)
(*b*)	$\displaystyle\sum_{r=2}^{n}\frac{2r-1}{r^2(r-1)^2}=\sum_{r=2}^{n}\frac{1}{(r-1)^2}-\frac{1}{r^2}$	M1
	$=\dfrac{1}{1^2}-\dfrac{1}{2^2}+\dfrac{1}{2^2}-\dfrac{1}{3^2}+\ldots+\dfrac{1}{(n-1)^2}-\dfrac{1}{n^2}$	M1
	$=1-\dfrac{1}{n^2}$ (*)	A1 cso (3)
		(5 marks)
2.	Identifying as critical values $-\frac{1}{2}$, $\frac{2}{3}$	B1, B1
	Establishing there are no further critical values	
	Obtaining $2x^2-2x+2$ or equivalent	M1
	$\Delta=4-16<0$	A1
	Using exactly two critical values to obtain inequalities	M1
	$-\frac{1}{2}<x<\frac{2}{3}$	A1
		(6 marks)
Graphical alt.	Identifying $x=-\frac{1}{2}$ and $x=\frac{2}{3}$ as vertical asymptotes	B1, B1
	Two rectangular hyperbolae oriented correctly with respect to asymptotes in the correct half-planes.	M1
	Two correctly drawn curves with no intersections	A1
	As above	M1, A1
		

((*) indicates final line is given on the paper; cso = correct solution only; ft = follow-through mark)

Question Number	Scheme	Marks
3. (*a*)	$x^3 - 27 = (x-3)(x^2+3x+9)$	M1
	($x = 3$ is one root). Others satisfy $(x^2 + 3x + 9) = 0$ (*)	A1 (2)
(*b*)	Roots are $x = 3$	B1
	and $x = \dfrac{-3 \pm \sqrt{9-36}}{2}$	M1
	$= -\dfrac{3}{2} + \dfrac{3\sqrt{3}}{2}\text{i},\ -\dfrac{3}{2} - \dfrac{3\sqrt{3}}{2}\text{i}$	A1 (3)
(*c*)	 3 and one other root in correct quad	B1
	Root in complex conjugate posn.	B1 ft (2)
		(7 marks)
4. (*a*)	f(1) = –4 f(2) = 1	M1
	Change of sign (and continuity) implies $\alpha \in (1, 2)$	A1 (2)
(*b*)	f(1.5) = –2.3… $\Rightarrow 1.5 < \alpha < 2$	B1
	f(1.75) = –0.9… $\Rightarrow 1.75 < \alpha < 2$	
	f(1.875) = –0.03… $\Rightarrow 1.875 < \alpha < 2$	B1 (2)
(*c*)	$f'(x) = 3^x \ln 3 - 1$	M1
	$f'(2) = 8.8875\ldots$ can imply previous M1	A1
	$\alpha_2 = 2 - \dfrac{1}{8.8875...}$ cao	M1, A1 **(4)**
		(8 marks)
	Incorrect differentiation can gain M0, A0, M1, A0	
	NB Exact answer is 1.8789…	
Alt to 4(*a*)	 Two graphs with single point of intersection ($x > 0$)	M1
	Two calculations at both $x = 1$ and $x = 2$	A1

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Question Number	Scheme		Marks
5. (*a*)	$\frac{4+3\text{i}}{2+4\text{i}} = \frac{(4+3\text{i})(2-4\text{i})}{20} = \frac{20-10\text{i}}{20}$ $(= 1 - \frac{1}{2}\text{i})$		M1
	$\lvert z \rvert = \sqrt{1^2 + (-\frac{1}{2})^2}$, $= \frac{\sqrt{5}}{2}$	awrt 1.12, accept exact equivalents	M1, A1 (3)
(*b*)	$\frac{(a+3\text{i})(2-a\text{i})}{(2+a\text{i})(2-a\text{i})} = \frac{5a+(6-a^2)\text{i}}{4+a^2}$	accept in (*a*) if clearly applied to (*b*)	M1
	$(\tan\frac{\pi}{4} =)\ 1 = \frac{6-a^2}{5a}$	obtaining quadratic or equivalent	M1 A1
	$a^2 + 5a - 6 = (a+6)(a-1)$		
	$a = -6, 1$		M1 A1
	Reject $a = -6$, wrong quadrant/$-\frac{3\pi}{4}$, $\Rightarrow$ one value		A1 (6)
			(9 marks)
Alt. (*a*)	$\lvert 4+3\text{i} \rvert = 5$, $\lvert 2+4\text{i} \rvert = \sqrt{20}$		M1
	$\lvert z \rvert = \frac{5}{\sqrt{20}}$ $(= \frac{\sqrt{5}}{2})$		M1 A1 (3)
(*b*)	$\arg z = \arg(a+3\text{i}) - \arg(2+a\text{i})$		
	$\frac{\pi}{4} = \arctan\frac{3}{a} - \arctan\frac{a}{2}$		M1
	$1 = \frac{\frac{3}{a} - \frac{a}{2}}{1 + \frac{3}{2}}$		M1 A1 (3)
	leading to $a^2 + 5a - 6 = 0$, then as before		

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Question Number	Scheme	Marks
6. (*a*)	$\frac{dt}{dx} = 2x$ or equivalent	M1
	$I = \frac{1}{2}\int te^{-t}\,dt$ complete substitution	M1
	$= -te^{-t}\,dt + \frac{1}{2}\int e^{-t}\,dt$	M1 A1
	$= -\frac{1}{2}te^{-t} - \frac{1}{2}e^{-t}\ (+c)$	A1
	$= -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2}\ (+c)$	A1 **(6)**
(*b*)	I.F. $= e^{\int\frac{3}{x}dx} = x^3$ (or multiplying equation by x^2)	B1
	$\frac{d}{dx}(x^3y) = x^3e^{-x^2}$ or $x^3y = \int x^3e^{-x^2}\,dx$	M1
	$x^3y = -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2} + \underline{C}$	A1ft <u>A1</u> (4)
		(10 marks)
Alts (*a*)	(i) mark $t = -x^2$ similarly	M1
	(ii) $\int x^2.(xe^{-x^2})\,dx$ with evidence of attempt at integration by parts	M1
	$= x^2(-\frac{1}{2}e^{-x^2}) + \frac{1}{2}\int 2x.e^{-x^2}\,dx$	M1 A1 + A1
	$= -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2}\ (+c)$	M1 A1 (6)
	(iii) $u = e^{-x^2}$, $\frac{du}{dx} = -2xe^{-x^2}$	M1
	$x^2 = \ln u$ hence $I = \int \frac{1}{2}\ln u\,du$	M1
	$= \frac{1}{2}u\ln u - \frac{1}{2}\int u.\frac{1}{u}\,du$	M1 A1
	$= \frac{1}{2}u\ln u - \frac{1}{2}u\ (+c)$	A1
	$= -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2}\ (+c)$	A1 (6)
	(The result $\int \ln u\,du = u\ln u - u$ may be quoted, gaining M1 A1 A1 but must be completely correct.)	

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Question Number	Scheme	Marks
7. (*a*)	A: $(5a, 0)$ B: $(3a, 0)$ allow on a sketch	B1, B1 (2)
(*b*)	$3 + 2\cos\theta = 5 - 2\cos\theta$	M1
	$\cos\theta = \frac{1}{2}$	M1
	$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ (allow $-\frac{\pi}{3}$)	A1
	Points are $(4a, \frac{\pi}{3})$, $(4a, \frac{5\pi}{3})$	A1 (4)
(*c*)	$(\frac{1}{2})\int r^2\,d\theta = (\frac{1}{2})\int (5 - 2\cos\theta)^2\,d\theta$	
	$= (\frac{1}{2})\int (25 - 20\cos\theta + 4\cos^2\theta)\,d\theta$	M1
	$= (\frac{1}{2})\int (25 - 20\cos\theta + 2\cos 2\theta + 2)\,d\theta$	M1
	$= (\frac{1}{2})[27\theta - 20\sin\theta + \sin 2\theta]$	A1
	$(\frac{1}{2})\int r^2\,d\theta = (\frac{1}{2})\int (3 + 2\cos\theta)^2\,d\theta$	
	$= (\frac{1}{2})\int (9 + 12\cos\theta + 4\cos^2\theta)\,d\theta$	
	$= (\frac{1}{2})\int (11 + 12\cos\theta + 2\cos 2\theta)\,d\theta$	
	$= (\frac{1}{2})[11\theta + 12\sin\theta + \sin 2\theta]$ 2nd integration	A1
	Area $= 2 \times \frac{1}{2}\int(5 - 2\cos\theta)^2\,d\theta + 2 \times \frac{1}{2}\int(3 + 2\cos\theta)^2\,d\theta$ (addition; condone 2/½)	M1
	$= \dots \int_0^{\frac{\pi}{3}} \dots + \dots \int_{\frac{\pi}{3}}^{\pi} \dots$ correctly identifying limits with ∫s	A1
	$= a^2[27 \times \frac{\pi}{3} - 10\sqrt{3} + \frac{\sqrt{3}}{2}] + a^2[11(\pi - \frac{\pi}{3}) - 6\sqrt{3} - \frac{\sqrt{3}}{2}]$	dM1
	$= a^2[49 - 48\sqrt{3}]$ (*)	A1 cso (8)
		(14 marks)

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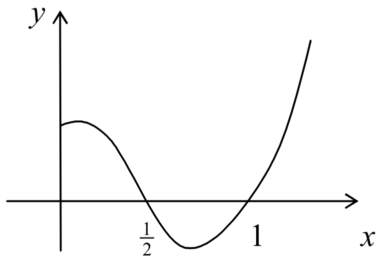
Question Number	Scheme	Marks
8. (*a*)	$y' = 2kt.e^{3t} + 3kt^2 e^{3t}$ use of product rule	M1
	$y'' = 2ke^{3t} + 12kt\,e^{3t} + 9t^2 e^{3t}$ product rule, twice	M1
	substituting $2k + 12kt + 9kt^2 - 12kt - 18kt^2 + 9kt^2 = 4$	M1
	$k = 2$	A1 (4)
(*b*)	Aux. eqn. (if used) $(m-3)^2 = 0$ $m = 3$, repeated	
	$y_{C.F.} = (A + Bt)\,e^{3t}$ M1 required form (allow just written down)	M1 A1
	G.S. $y = (A + Bt)\,e^{3t} + 2t^2 e^{3t}$ (ft on $2t^2 e^{3t}$)	A1 ft (3)
(*c*)	$t = 0, y = 3 \Rightarrow A = 3$	B1
	$y' = Be^{3t} + 3(A + Bt)\,e^{3t} + 4te^{3t} + 6t^2e^{3t}$	M1
	$y' = 0, t = 0 \Rightarrow 1 = B + 3A \Rightarrow B = -8$	M1
	$y = (3 - 8t + 2t^2)e^{3t}$	A1 (4)
(*d*)		
	$\cup$ shape crossing +ve x-axis	B1
	$\frac{1}{2}$, 1	B1
	$y' = (-3 + 4t)e^{3t} + 3(1 - 3t + 2t^2)e^{3t} = 0$	
	$6t^2 - 5t = 0$	M1
	$t = \dfrac{5}{6}$	A1
	$y = -\dfrac{1}{9}e^{2.5}$ (≈ -1.35) awrt –1.35	A1 (5)
		(16 marks)

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