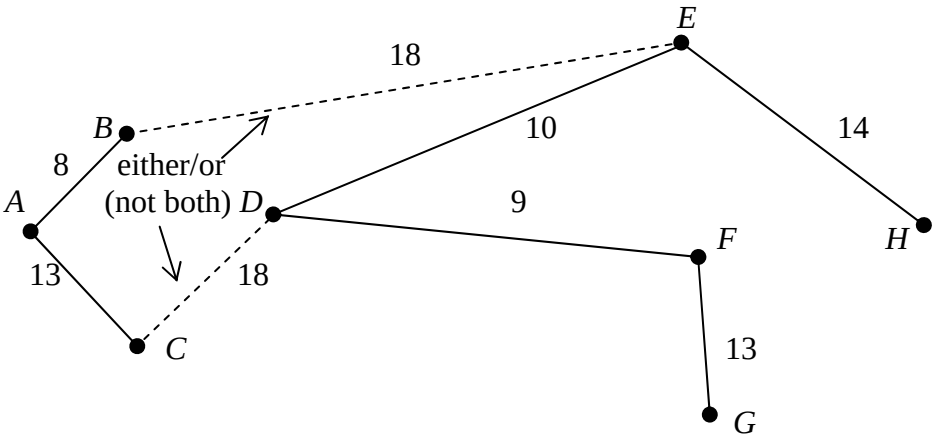
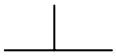
Question Number	Scheme	Marks
1. (*a*)		
		B2, 1, 0 (2)
(*b*)	e.g. Let v = value of the game, p = pay-off, q_i = P(B plays i), = 1, 2, 3	
	Matrix becomes	B1
	maximise $p = v$	
	subject to $v - 9q_1 - 4q_2 - q_3 + r = 0$	M1
	$v - 2q_1 - 7q_2 - 10q_3 + s = 0$	A2 ft, 1 ft, 0
	$q_1 + q_2 + q_3 + t = 0$	(4)
		(6 marks)

	A(I)	A(II)
B(I)	3	–4
B(II)	–2	1
B(III)	–5	4

	A(I)	A(II)
B(I)	9	2
B(II)	4	7
B(III)	1	10

(ft = follow through mark)

Question Number		Scheme	Marks
2.	(*a*)	In the *practical* TSP each vertex must be visited *at least once*	B1
		In the *classical* TSP each vertex must be visited *exactly once*	B1 (2)
	(*b*)	*AB*, *DF*, *DE*, (reject *EF*), $\left\{\begin{matrix} FG \\ AC \end{matrix}\right\} EH \left\{\begin{matrix} DC \\ \text{or} \\ BE \end{matrix}\right\}$	M1 A1
			B1 (3)
	(*c*)	Initial upper bound = 2 × 85 = 170 km	M1 A1 (2)
	(*d*)	e.g. when *CD* is part of the tree	
		Use *GH* (saving 26) and *BD* (saving 19) giving new upper bound of 125 km	M1 A1
		Tour *A B D E H G F D C A*	A1 (3)
		(or e.g. when *BE* is part of the tree, use *CG* (saving 40) giving new upper bound of 130 km; Tour *A B E H E D F G C A*)	**(10 marks)**

Question Number	Scheme	Marks
3. (*a*)(i)	**Either** *rows then columns* giving |	I	II	III	IV | |---|---|---|---|---|---| | *C* | 0 | 22 | 16 | 4 | | *J* | 1 | 20 | 24 | 0 | | *N* | 1 | 18 | 18 | 0 | | *S* | 1 | 23 | 26 | 0 | then |	I	II	III	IV | |---|---|---|---|---| | *C* | 0 | 4 | 0 | 4 | | *J* | 1 | 2 | 8 | 0 | | *N* | 1 | 0 | 2 | 0 | | *S* | 1 | 5 | 10 | 0 |	M1, A1, A1 (3)
	3 lines only needed  least element 1 so |	I	II	III	IV	 |---|---|---|---|---| | *C* | 0 | 4 | 0 | 5 | | *J* | 0 | 1 | 7 | 0 | | *N* | 1 | 0 | 2 | 1 | | *S* | 0 | 4 | 9 | 0 |	M1, A1, A1 (3)
alt (*a*)(i)	**or** *columns then rows* giving |	I	II	III	IV	 |---|---|---|---|---| | *C* | 1 | 2 | 0 | 6 | | *J* | 2 | 0 | 8 | 2 | | *N* | 4 | 0 | 4 | 4 | | *S* | 0 | 1 | 8 | 0 | (then no change)	M1 A1
	3 lines only needed  and *either* row 1 *or* column 3 if row 1 least uncovered 2 |	I	II	III	IV	 |---|---|---|---|---| | *C* | 1 | 4 | 0 | 6 | | *J* | 0 | 0 | 6 | 0 | | *N* | 2 | 0 | 2 | 2 | | *S* | 0 | 3 | 8 | 0 |	M1 A3 (6)
	if column 3 least uncovered 1 |	I	II	III	IV | |---|---|---|---|---| | *C* | 0 | 2 | 0 | 5 | | *J* | 1 | 0 | 8 | 1 | | *N* | 3 | 0 | 4 | 3 | | *S* | 0 | 2 | 9 | 0 | then I least uncoverd 1 |	I	II	III	IV | |---|---|---|---|---| | *C* | 0 | 3 | 0 | 5 | | *J* | 0 | 0 | 7 | 0 | | *N* | 2 | 0 | 3 | 2 | | *S* | 0 | 3 | 9 | 0 |	M1 A3 (6)
(*a*)(ii)	*C* – III, *J* – I or IV, *N* – II, *S* – IV or I 83 minutes ∴ 11.23 a.m. (*continued page 4*)	M1 A1 M1 A1 (4)

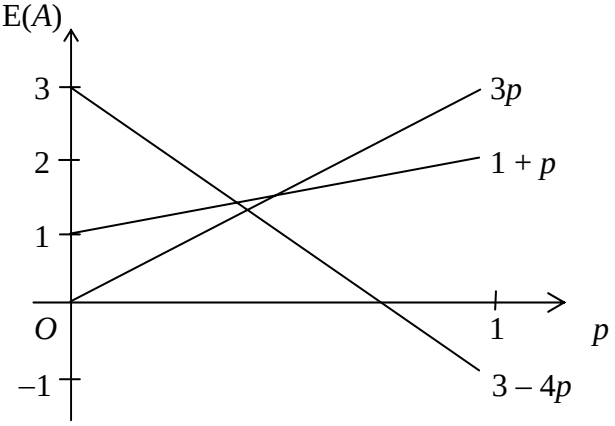
<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>3. (b)</td><td>Subtracting all entries from some $n \geq 36$ (stated)

e.g. subtractions from 36

<table><tr><th></th><th>I</th><th>II</th><th>III</th><th>IV</th></tr><tr><td>C</td><td>24</td><td>2</td><td>8</td><td>20</td></tr><tr><td>J</td><td>23</td><td>4</td><td>0</td><td>24</td></tr><tr><td>N</td><td>21</td><td>4</td><td>4</td><td>22</td></tr><tr><td>S</td><td>25</td><td>3</td><td>0</td><td>26</td></tr></table></td><td>M1

A2, 1, 0 (3)

(13 marks)</td></tr>
</table>

Question Number	Scheme	Marks
4. (*a*)	Player *A*: row minimums are –1, 0, –3 so maximin choice is play II	M1 A1
	Player *B*: column maximums are 2, 3, 3 so minimax choice is play I	M1 A1 (4)
(*b*)	Since *A*'s maximin (0) ≠ *B*'s minimax (2) there is no stable solution	B1 (1)
(*c*)	For player *A* row II dominates row III, so *A* will *now* play III	B2, 1, 0 (2)
(*d*)	Let *A* play I with probability p and II with probability $(1 - p)$	
	If *B* plays I, *A*'s expected winnings are $2p + (1 - p) = 1 + p$	
	If *B* plays II, *A*'s expected winnings are $-p + 3(1 - p) = 3 - 4p$	M1, A2, 1, 0
	If *B* plays III, *A*'s expected winnings are $3p$	(3)
	E(*A*) 	M1
	$3 - 4p = 3p \Rightarrow p = \frac{3}{7}$	A1
	A should play I with probability $\frac{3}{7}$	
	A should play II with probability $\frac{4}{7}$	A1
	and never play III	
	The value of the game is $\frac{9}{7}$ to *A*	A1 (4)
		(14 marks)

(ft = follow through mark)

Question Number	Scheme	Marks
5. (*a*)	$\begin{array}{c\|ccc} & D & E & F \\ \hline A & 6 & & \\ B & 0 & 5 & \\ C & & 4 & 4 \end{array}$ *or* $\begin{array}{c\|ccc} & D & E & F \\ \hline A & 6 & 0 & \\ B & & 5 & \\ C & & 4 & 4 \end{array}$	M1 A1
	cost £470	A1 (3)
(*b*)	$S_A = 0, S_B = 0, S_C = -10$	$S_A = 0, S_B = -10, S_C = -20$
	$D_D = 20, D_E = 30, D_F = 40$	$D_D = 20, D_E = 40, D_F = 50$ M1 A1
	$I_{AE} = 40 - 30 = 10$	$I_{AF} = 10 - 50 = -40$
	$I_{AF} = 10 - 40 = -30$	$I_{BD} = 20 - 10 = 10$
	$I_{BF} = 40 - 40 = 0$	$I_{BF} = 40 - 40 = 0$
	$I_{CD} = 10 - 10 = 0$	$I_{CD} = 10 - 0 = 10$ M1 A1 (4)
	Choose AF as entering route	
	$AF(+) \rightarrow CF(-) \rightarrow CE(+) \rightarrow BE(-) \rightarrow BD(+) \rightarrow AD(-)$	$AF(+) \rightarrow CF(-) \rightarrow CE(+) \rightarrow AE(-)$
	Exiting route CF $\theta = 4$	Exiting route AE $\theta = 0$ M1 A1 ft
	$\begin{array}{c\|ccc} & D & E & F \\ \hline A & 2 & & 4 \\ B & 4 & 1 & \\ C & & 8 & \end{array}$	$\begin{array}{c\|ccc} & D & E & F \\ \hline A & 6 & & 0 \\ B & & 5 & \\ C & & 4 & 4 \end{array}$ A1
	$S_A = 0, S_B = 0, S_C = -10$	$S_A = 0, S_B = 10, S_C = 20$
	$D_D = 20, D_E = 30, D_F = 40$	$D_D = 20, D_E = 0, D_F = 10$
	$I_{AE} = 10, \quad I_{BF} = 30,$	$I_{AE} = 40, \quad I_{BD} = -10,$
	$I_{CD} = 0, \quad I_{CF} = 30$	$I_{BF} = 20, \quad I_{CD} = -30$ M1 A1 A1
		$CD(+) \rightarrow AD(-) \rightarrow AF(+) \rightarrow CF(-)$
		$\theta = 4$
		$\begin{array}{c\|ccc} & D & E & F \\ \hline A & 2 & & 4 \\ B & & 5 & \\ C & 4 & 4 & \end{array}$
		$S_A = 0, S_B = 0, S_C = -10$
		$D_D = 20, D_E = 30, D_F = 10$
		$I_{AE} = 40, I_{BD} = 0, I_{BF} = 30, I_{CF} = 30$
	$\therefore$ optimal, cost £350	$\therefore$ optimal, cost £350 A1 (7)
		(14 marks)

6. (*a*) Total cost = 2 × 40 + 350 + 200 = £630 — M1 A1 (2)

(*b*)

Stage	Demand	State	Action	Destination	Value	
(2)	(5)	(1)	(4)	(0)	(590 + 200 = 790)	
		(2)	(3)	(0)	280 + 200 = 480	M1 A1
			(4)	(1)	630 + 240 = 870	
		(3)	(2)	0	320 + 200 = 520	M1 A1 (4)
			3	1	320 + 240 = 560	
			4	2	670 + 80 = 750	
3	3	0	4	1	550 + 790 = 1340	M1 A1
		1	3	1	240 + 790 = 1030	M1 A1 ft
			4	2	590 + 480 = 1070	
4	3	0	3	0	200 + 1340 = 1540	M1 A1 ft (6)
			4	1	550 + 1030 = 1580	

Month	August	September	October	November
Make	3	4	4	2

M1 A1

cost = £1540 — A1 ft (3)

Profit per cycle = 13 × 1400 = 18 200 — Cost of Kim's time = £2000

Cost of production = £1540 — B1

∴ Toatl profit = 18 200 – 3540 — M1

= 14 660 — A1 ft (3)

(18 marks)

(ft = follow through mark)