Paper Reference(s)

6680

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Wednesday 22 January 2003 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions. Pages 6, 7 and 8 are blank.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. A boy enters a large horizontal field and sees a friend 100 m due north. The friend is walking in an easterly direction at a constant speed of 0.75 m s^{-1}. The boy can walk at a maximum speed of 1 m s^{-1}.

Find the shortest time for the boy to intercept his friend and the bearing on which he must travel to achieve this.

(6)

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2. Boat *A* is sailing due cast at a constant speed of 10 km h^{-1}. To an observer on *A*, the wind appears to be blowing from due south. A second boat *B* is sailing due north at a constant speed of 14 km h^{-1}. To an observer on *B*, the wind appears to be blowing from the south west. The velocity of the wind relative to the earth is constant and is the same for both boats.

Find the velocity of the wind relative to the earth, stating its magnitude and direction.

(7)

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3. A small pebble of mass m is placed in a viscous liquid and sinks vertically from rest through the liquid. When the speed of the pebble is v the magnitude of the resistance due to the liquid is modelled as mkv^2, where k is a positive constant.

Find the speed of the pebble after it has fallen a distance D through the liquid.

(11)

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4. **Figure 1**

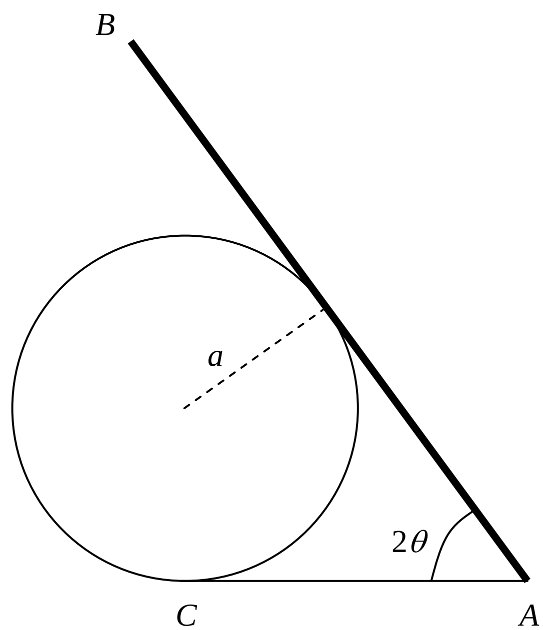


Figure 1 shows a uniform rod AB, of mass m and length $4a$, resting on a smooth fixed sphere of radius a. A light elastic string, of natural length a and modulus of elasticity $\frac{3}{4}mg$, has one end attached to the lowest point C of the sphere and the other end attached to A. The points A, B and C lie in a vertical plane with $\angle BAC = 2\theta$, where $\theta < \frac{\pi}{4}$.

Given that AC is always horizontal,

(*a*) show that the potential energy of the system is

$$\frac{mga}{8}(16 \sin 2\theta + 3 \cot^2\theta - 6 \cot \theta) + \text{constant},$$

(7)

(b) show that there is a value of θ for which the system is in equilibrium such that $0.535 < \theta < 0.545$.

(6)

(c) Determine whether this position of equilibrium is stable or unstable.

(3)

5. A particle P moves in a straight line. At time t seconds its displacement from a fixed point O on the line is x metres. The motion of P is modelled by the differential equation

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} + 2x = 12\cos 2t - 6\sin 2t.$$

When $t = 0$, P is at rest at O.

(*a*) Find, in terms of t, the displacement of P from O.

(11)

(*b*) Show that P comes to instantaneous rest when $t = \frac{\pi}{4}$.

(2)

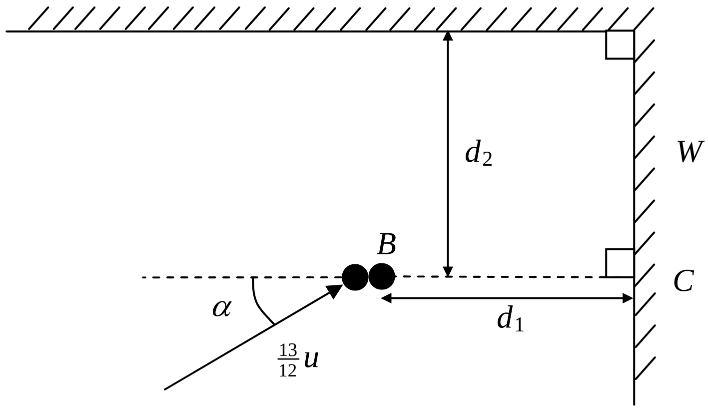
(*c*) Find, in metres to 3 significant figures, the displacement of P from O when $t = \frac{\pi}{4}$.

(2)

(*d*) Find the approximate period of the motion for large values of t.

(2)

6. **Figure 2**



A small ball Q of mass $2m$ is at rest at the point B on a smooth horizontal plane. A second small ball P of mass m is moving on the plane with speed $\frac{13}{12}u$ and collides with Q. Both the balls are smooth, uniform and of the same radius. The point C is on a smooth vertical wall W which is at a distance d_1 from B, and BC is perpendicular to W. A second smooth vertical wall is perpendicular to W and at a distance d_2 from B. Immediately before the collision occurs, the direction of motion of P makes an angle α with BC, as shown in Fig. 2, where $\tan \alpha = \frac{5}{12}$. The line of centres of P and Q is parallel to BC. After the collision Q moves towards C with speed $\frac{3}{5}u$.

(*a*) Show that, after the collision, the velocity components of P parallel and perpendicular to CB are $\frac{1}{5}u$ and $\frac{5}{12}u$ respectively.

(4)

(*b*) Find the coefficient of restitution between P and Q.

(2)

(*c*) Show that when Q reaches C, P is at a distance $\frac{4}{3}d_1$ from W.

(3)

For each collision between a ball and a wall the coefficient of restitution is $\frac{1}{2}$.

Given that the balls collide with each other again,

(*d*) show that the time between the two collisions of the balls is $\dfrac{15d_1}{u}$,

(4)

(*e*) find the ratio $d_1 : d_2$.

(5)

END