Paper Reference(s)

6672

Edexcel GCE

Pure Mathematics P2

Advanced/Advanced Subsidiary

Tuesday 5 November 2002 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P2), the paper reference (6672), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions. Pages 7 and 8 are blank.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. Express $\dfrac{y+3}{(y+1)(y+2)} - \dfrac{y+1}{(y+2)(y+3)}$ as a single fraction in its simplest form.

(5)

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2. (*a*) Using the substitution $u = 2^x$, show that the equation $4^x - 2^{(x+1)} - 15 = 0$ can be written in the form $u^2 - 2u - 15 = 0$.

(2)

(*b*) Hence solve the equation $4^x - 2^{(x+1)} - 15 = 0$, giving your answers to 2 decimals places.

(4)

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3. (*a*) Express $1.5 \sin 2x + 2 \cos 2x$ in the form $R \sin (2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving your values of R and α to 3 decimal places where appropriate.

(4)

(*b*) Express $3 \sin x \cos x + 4 \cos^2 x$ in the form $a \cos 2x + b \sin 2x + c$, where a, b and c are constants to be found.

(2)

(*c*) Hence, using your answer to part (*a*), deduce the maximum value of $3 \sin x \cos x + 4 \cos^2 x$.

(2)

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4. The sequence $u_1, u_2, u_3, \ldots, u_n$ is defined by the recurrence relation

$$u_{n+1} = pu_n + 5,\ u_1 = 2, \text{ where } p \text{ is a constant.}$$

Given that $u_3 = 8$,

(*a*) show that one possible value of p is $\frac{1}{2}$ and find the other value of p.

(5)

Using $p = \frac{1}{2}$,

(*b*) write down the value of $\log_2 p$.

(1)

Given also that $\log_2 q = t$,

(*c*) express $\log_2 \left(\dfrac{p^3}{\sqrt{q}}\right)$ in terms of t.

(3)

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5. The curve C with equation $y = p + qe^x$, where p and q are constants, passes through the point $(0, 2)$. At the point $P(\ln 2, p + 2q)$ on C, the gradient is 5.

(*a*) Find the value of p and the value of q.

(5)

The normal to C at P crosses the x-axis at L and the y-axis at M.

(*b*) Show that the area of $\triangle OLM$, where O is the origin, is approximately 53.8.

(3)

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6. **Figure 1**

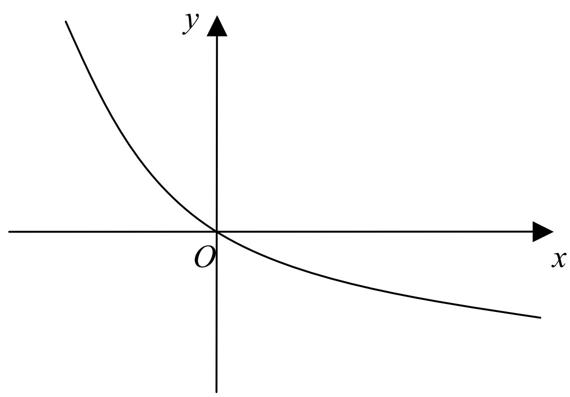

Figure 1 shows a sketch of the curve with equation $y = e^{-x} - 1$.

(*a*) Copy Fig. 1 and on the same axes sketch the graph of $y = \frac{1}{2}|x - 1|$. Show the coordinates of the points where the graph meets the axes.

(2)

The x-coordinate of the point of intersection of the graph is α.

(*b*) Show that $x = \alpha$ is a root of the equation $x + 2e^{-x} - 3 = 0$.

(3)

(*c*) Show that $-1 < \alpha < 0$.

(2)

The iterative formula $x_{n+1} = -\ln[\frac{1}{2}(3 - x_n)]$ is used to solve the equation $x + 2e^{-x} - 3 = 0$.

(*d*) Starting with $x_0 = -1$, find the values of x_1 and x_2.

(2)

(*e*) Show that, to 2 decimal places, $\alpha = -0.58$.

(2)

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7. **Figure 2**

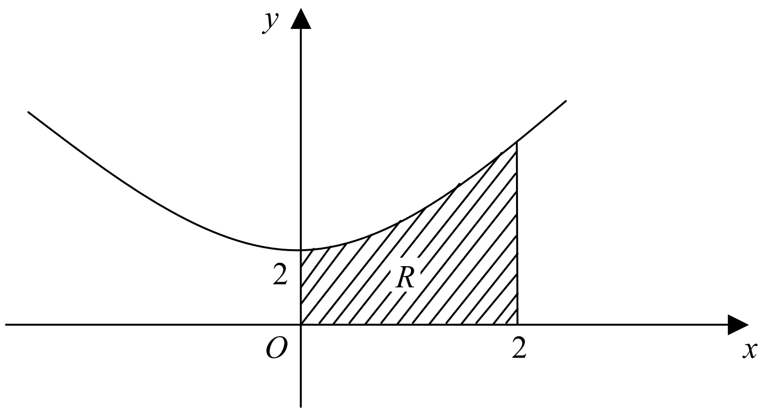

Figure 2 shows part of the curve with equation $y = x^2 + 2$.

The finite region R is bounded by the curve, the x-axis and the lines $x = 0$ and $x = 2$.

(*a*) Use the trapezium rule with 4 strips of equal width to estimate the area of R.

(5)

(*b*) State, with a reason, whether your answer in part (*a*) is an under-estimate or over-estimate of the area of R.

(1)

(*c*) Using integration, find the volume of the solid generated when R is rotated through 360° about the x-axis, giving your answer in terms of π.

(6)

8. The function f is defined by f: $x \mapsto \dfrac{3x-1}{x-3}$, $x \in \mathbb{R}$, $x \neq 3$.

(a) Prove that $f^{-1}(x) = f(x)$ for all $x \in \mathbb{R}$, $x \neq 3$.

(3)

(b) Hence find, in terms of k, ff(k), where $x \neq 3$.

(2)

Figure 3

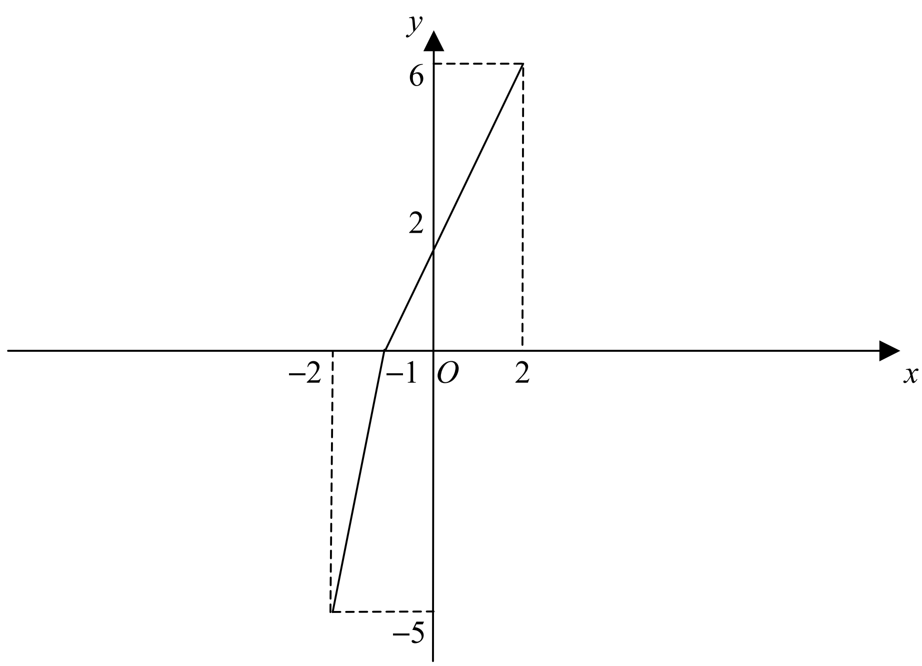

Figure 3 shows a sketch of the one-one function g, defined over the domain $-2 \leq x \leq 2$.

(c) Find the value of fg(−2).

(3)

(d) Sketch the graph of the inverse function g^{-1} and state its domain.

(3)

The function h is defined by h: $x \mapsto 2g(x-1)$.

(e) Sketch the graph of the function h and state its range.

(3)

END