1. (a) Impulse = change in linear momentum

$I = m\,V \;\Rightarrow\; V = \dfrac{I}{m}$ M1A1

Moment of impulse = change in ang. mom.

🗁 $a\,I$ = 🗎 $ma^2\,\omega$; $\Rightarrow\; a\omega = \dfrac{3I}{2m}$ (or ω = ..) M1A1;A1

Speed of $B = V + a\omega$, ; $= \boldsymbol{\dfrac{5I}{2m}}$ M1;A1 **(7)**

(b) Valid method for x [e.g. $V \pm x\,\omega = 0$]; $x = \dfrac{V}{\omega} = \dfrac{I}{m}\cdot\dfrac{2ma}{3I}$ M1;A1√

[A1√ dep. on M_1, M_2 and M_4]

= ✍ ***a*** A1 **(3)**

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2 $\ddot{y} = 0 \;\Rightarrow\; \dot{y} = u \;\Rightarrow\; y = u\,t$ M1A1

[M for integration but accept with no working]

$\ddot{x} = -4\,y$; $(= -4\,u\,t)$ B1

$\Rightarrow\; \dot{x} = -2\,u\,t^2 + c$ M1A√

[M for $\int(-4y)\,dt$ *with* $y = f(t)$]

Using limits correctly or finding "c" $(\dot{x} = -2\,u\,t^2 + u)$ M1

Integrating: $x = u\,t -$ ✍ $u\,t^3$ [M dep. on prev. two Ms] M1A1

Setting x = 0 and solving for t M1

$\boldsymbol{t} = \sqrt{\dfrac{3}{2}}$ **s or 1.22 s** A1 **[10]**

3. (a) $\frac{1}{r}\frac{\mathrm{d}}{\mathrm{dt}}\left(r^2\dot{\theta}\right) = 0$; $\Rightarrow$ $r^2\dot{\theta}$ = constant = h M1A1 **(2)**

(b) $r = a\sec 3\theta$, $-\frac{\pi}{6} < \theta < \frac{\pi}{6}$

$\dot{r} = 3a\sec 3\theta\tan 3\theta.\dot{\theta}$ B1

Using (a) to eliminate $\dot{\theta}$

$= 3r\tan 3\theta.\frac{h}{r^2} = \frac{3h\tan 3\theta}{r}$ (or $\frac{3ah\sec 3\theta\tan 3\theta}{r^2}$ or $\frac{3h\sin 3\theta}{a}$) M1

$\ddot{r} = 3h\frac{(r.3\sec^2 3\theta.\dot{\theta} - \tan 3\theta.\dot{r})}{r^2}$ or equivalent M1A1A1

[OR $\ddot{r} = 3a\sec 3\theta\tan 3\theta\ddot{\theta} + [9a\sec 3\theta\tan^2 3\theta + 9a\sec^2 3\theta]\dot{\theta}^2$ M1A2,1,0

Eliminate $\dot{\theta}$ *and* $\ddot{\theta}$ M1]

Complete method to stage $\ddot{r} = f(r)$ *only* M1

[e.g. $= \frac{3h}{r^2}(3\frac{h}{r}\sec^2 3\theta - 3\frac{h}{r}\tan^2 3\theta) = 3\frac{h}{r^2}\ .\ 3\frac{h}{r}(\sec^2 3\theta - \tan^2 3\theta)$]

$\ddot{r} = 9\frac{h^2}{r^3}$ A1

Mag. of accel. = $\ddot{r} - r\dot{\theta}^2$; $= 9\frac{h^2}{r^3} - \frac{h^2}{r^3} = 8\frac{h^2}{r^3}$ (**$k = 8h^2$**) M1A1√ **(9)**

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4. (a) [s = 20 sin ψ , $\frac{ds}{d\varphi} = 20\cos\psi$ (i), $\dot{s}$ = 20 cos $\psi\dot{\psi}$ (ii)]

Equation of motion approach **:** – (m)g sinψ = (m)$\ddot{s}$ or (m) v $\frac{dv}{ds}$ M1A1

Arranging to integrable form: [A1√ on omission of – in prev. equation] M1A1√

[e.g. $-\int \frac{g}{20}s\ ds = \int v\,dv$ or $-20g\int \sin\psi\ \cos\ \psi\ d\psi = \int v\,dv$]

Integrate: $-\frac{g}{40}s^2$ = 🗀 v^2+ (C) or 5g cos 2ψ = 🗀 v^2 + (C) or equiv. M1A1√

Using limits correctly or finding C M1

$\Rightarrow 98\cos 2\psi + 49 = v^2$ * A1 **(8)**

[Alternative: Energy approach

🗀 $m\,v^2 = mg$ (7.5 – 'y') M1A1

Finding y

$\frac{dy}{ds} = \sin\ \psi = \frac{s}{20}$ or $\frac{dy}{d\psi} = 20\sin\psi\cos\psi$ M1A1

$\Rightarrow y = \frac{s^2}{40}$ (+ C) or y = – 5 cos 2ψ (+ C) or equivalent M1A1

Using limits correctly M1

$V^2 = 49 + 98\cos 2\psi$ * (cso)] A1

(b) Along normal: R – 100 g (cos ψ) = 100 $\frac{v^2}{\rho}$ M1A1

$\rho = \frac{ds}{d\psi} = 20\cos\ \psi$ B1

R = 100g + (5x147) = **1715 N** (1720,1700) A1 **(4)**

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5. (a) Method to find masses of end "discs" and curved "shell" M1

[$M = \rho(2\pi a^2 + 8\pi a^2)$, Masses are $\frac{M}{10}$, $\frac{M}{10}$ and $\frac{8M}{10}$]

$I = 2\left(\frac{1}{2}m_1 a^2\right) + m_2 a^2$ M1A1

$= \frac{9}{10}Ma^2$ * (cso) A1 **(4)**

(b) Energy approach:

KE terms $\frac{1}{2}Mv^2$, $\frac{1}{2}I(\dot{\theta})^2$ B1B1

(Loss in) PE $= Mg\,5a \sin\alpha$ M1

$v = a\dot{\theta}$ (seen anywhere) B1

Energy equation: $\frac{1}{2}Mv^2 + \frac{1}{2}I(\dot{\theta})^2$ = loss in PE M2

(no term in F)

Substituting for v and I (dep. on M2) M1

$\frac{1}{2}Ma^2(\dot{\theta})^2 + \frac{1}{2}\frac{9}{10}Ma^2(\dot{\theta})^2 = Mg\,5a\sin\alpha$ A1

$\frac{19}{20}Ma^2(\dot{\theta})^2 = Mg\,5a\sin\alpha$; $\dot{\theta} = 10\sqrt{\frac{g\sin\alpha}{19a}}$ M1;A1 **(10)**

[Alternative approach: **[14]**

$Mg\sin\alpha - F = M\ddot{x}$; $Fa = I\ddot{\theta}$ M1A1;M1

$\ddot{x} = a\ddot{\theta}$ (seen anywhere) B1

Method for $\ddot{x}$: $Mg\sin\alpha - \frac{9}{10}M\ddot{x} = M\ddot{x} \Rightarrow \ddot{x} = \frac{10}{19}g\sin\alpha$ M1A1

[M dep. on prev. two Ms]

$v^2 = 2\ddot{x}s \Rightarrow v^2 = \frac{100}{19}ga\sin\alpha$ M1A1√

$\dot{\theta} = \frac{v}{a} = 10\sqrt{\frac{g\sin\alpha}{19a}}$ M1A1]

6. (a) Vertical component of velocity at wall unchanged

$\uparrow\ 0 = 14 \sin\theta\ t - □\ g\, t^2$ M1A1

$= 14\left(\frac{3}{5}\right)t - 4.9\, t^2$

$\Rightarrow\ t = \frac{12}{7}$ s or 1.71 s or $\frac{84}{5g}$ M1A1 **(4)**

(b) Time to wall: $4 = 14\cos\theta\ t$; $t = \frac{5}{14}$ (0.357) M1;A1

Complete method for time from wall to A: $T = \frac{12}{7} - \frac{5}{14} = \frac{19}{14}$ (1.36) M1

Horizontal component of velocity = $14\cos\theta$ B1

Horizontal component of velocity after rebound = □ $(14\cos\theta)$ M1

Distance from wall to A = □ $(14\cos\theta)$ x $\frac{19}{14}$ = **7.6 s** M1A1 **(7)**

[Marks can be awarded if seen in longer methods for (a)]

(c) Vertical component of velocity at A is $14\sin\theta$ B1

After hitting ground $v\uparrow$ = □ $(14\sin\theta) = 4.2$ (V) B1√

Time from A to B: $0 = V\,t - □\ g\,t^2$; $t = \frac{6}{7}$ s (0.86) M1A1√;A1

Distance AB = □ $(14\cos\theta)$ x $\left(\frac{6}{7}\right)_C$ = **4.8 m** M1A1 **(7)**

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