Paper Reference(s)

6681

Edexcel GCE

Mechanics M5

Advanced/Advanced Subsidiary

Tuesday 18 June 2002 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions. Pages 6, 7 and 8 are blank.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

A small smooth ring of mass 0.5 kg moves along a smooth horizontal wire. The only forces acting on the ring are its weight, the normal reaction from the wire, and a constant force $(5\mathbf{i} + \mathbf{j} - 3\mathbf{k})$ N. The ring is initially at rest at the point with position vector $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ m, relative to a fixed origin.

Find the speed of the ring as it passes through the point with position vector $(3\mathbf{i} + \mathbf{k})$ m.

(5)

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2. Three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a rigid body. $\mathbf{F}_1 = (2\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ N, $\mathbf{F}_2 = (\mathbf{i} + \mathbf{j} - 4\mathbf{k})$ N and $\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j} + r\mathbf{k})$ N, where p, q and r are constants. All three forces act through the point with position vector $(3\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ m, relative to a fixed origin. The three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are equivalent to a single force $(5\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ N, acting at the origin, together with a couple **G**.

(*a*) Find the values of p, q and r.

(3)

(*b*) Find **G**.

(3)

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3. At time t seconds, the position vector of a particle P is $\mathbf{r}$ metres, relative to a fixed origin. The particle moves in such a way that

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} - 4\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \mathbf{0}.$$

At $t = 0$, P is moving with velocity $(8\mathbf{i} - 6\mathbf{j})$ m s^{-1}.

Find the speed of P when $t = \frac{1}{2}\ln 2$.

(7)

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4. A uniform plane lamina of mass m is in the shape of an equilateral triangle of side $2a$. Find, using integration, the moment of inertia of the lamina about one of its edges.

(9)

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5. A rocket is launched vertically upwards from rest. Initially, the total mass of the rocket and its fuel is 1000 kg. The rocket burns fuel at a rate of 10 kg s^{-1}. The burnt fuel is ejected vertically downwards with a speed of 2000 m s^{-1} relative to the rocket, and burning stops after one minute. At time t seconds, $t \leq 60$, after the launch, the speed of the rocket is v m s^{-1}. Air resistance is assumed to be negligible.

(*a*) Show that

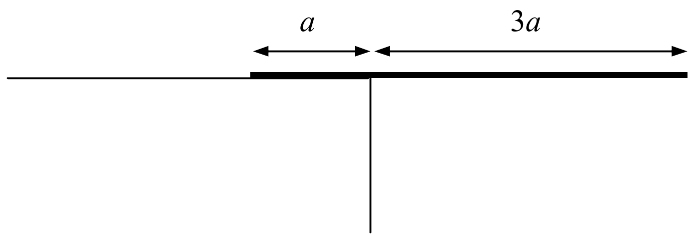
$$-9.8(100 - t) = (100 - t)\,\frac{\mathrm{d}v}{\mathrm{d}t} - 2000.$$

(8)

(*b*) Find the speed of the rocket when burning stops.

(6)

6. **Figure 1**



A rough uniform rod, of mass m and length $4a$, is rod is held on a rough horizontal table. The rod is perpendicular to the edge of the table and a length $3a$ projects horizontally over the edge, as shown in Fig. 1.

(a) Show that the moment of inertia of the rod about the edge of the table is $\frac{7}{3}ma^2$.

(2)

The rod is released from rest and rotates about the edge of the table. When the rod has turned through an angle θ, its angular speed is $\dot{\theta}$. Assuming that the rod has not started to slip,

(b) show that $\dot{\theta}^2 = \dfrac{6g\sin\theta}{7a}$,

(3)

(c) find the angular acceleration of the rod,

(2)

(d) find the normal reaction of the table on the rod.

(4)

The coefficient of friction between the rod and the edge of the table is μ.

(e) Show that the rod starts to slip when $\tan\theta = \frac{4}{13}\mu$

(6)

7. A uniform plane circular disc, of mass m and radius a, hangs in equilibrium from a point B on its circumference. The disc is free to rotate about a fixed smooth horizontal axis which is in the plane of the disc and tangential to the disc at B. A particle P, of mass m, is moving horizontally with speed u in a direction which is perpendicular to the plane of the disc. At time $t = 0$, P strikes the disc at its centre and adheres to the disc.

(a) Show that the angular speed of the disc immediately after it has been struck by P is $\dfrac{4u}{9a}$.

(6)

It is given that $u^2 = \frac{1}{10}ag$, and that air resistance is negligible.

(b) Find the angle through which the disc turns before it first comes to instantaneous rest.

(5)

The disc first returns to its initial position at time $t = T$.

(c) (i) Write down an equation of motion for the disc.

(ii) Hence find T in terms of a, g and m, using a suitable approximation which should be justified.

(6)

END