Paper Reference(s)

6680

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Monday 24 June 2002 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has six questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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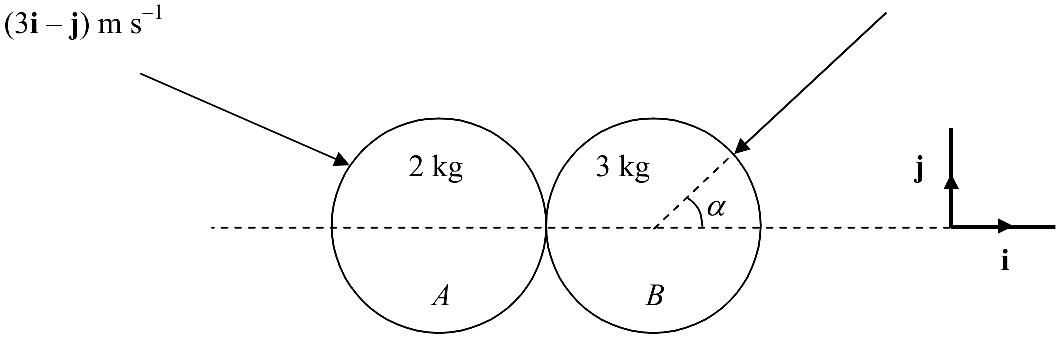
1. **Figure 1**



Two smooth uniform spheres *A* and *B*, of equal radius, are moving on a smooth horizontal plane. Sphere *A* has mass 2 kg and sphere *B* has mass 3 kg. The spheres collide and at the instant of collision the line joining their centres is parallel to **i**. Before the collision *A* has velocity (3**i** – **j**) m s^{-1} and after the collision it has velocity (–2**i** – **j**) m s^{-1}. Before the collision the velocity of *B* makes an angle α with the line of centres, as shown in Fig. 1, where tan $\alpha = 2$. The coefficient of restitution between the spheres is $\frac{1}{2}$.

Find, in terms of **i** and **j**, the velocity of *B* before the collision.

(9)

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2. Ship *A* is steaming on a bearing of 060° at 30 km h^{-1} and at 9 a.m. it is 20 km due west of a second ship *B*. Ship *B* steams in a straight line.

(*a*) Find the least speed of *B* if it is to intercept *A*.

(3)

Given that the speed of *B* is 24 km h^{-1},

(*b*) find the earliest time at which it can intercept *A*.

(7)

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3. The engine of a car of mass 800 kg works at a constant rate of 32 kW. The car travels along a straight horizontal road and the resistance to motion of the car is proportional to the speed of the car. The car starts from rest and t seconds later it has a speed of v m s^{-1}.

(*a*) Show that

$$800v\frac{\mathrm{d}v}{\mathrm{d}t} = 32\,000 - kv^2, \text{ where } k \text{ is a positive constant.}$$

(3)

Given that the limiting speed of the car is 40 m s^{-1}, find

(*b*) the value of k,

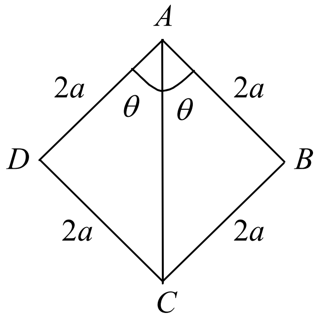
(2)

(*c*) v in terms of t.

(7)

4.

Figure 2



Four identical uniform rods, each of mass m and length $2a$, are freely jointed to form a rhombus $ABCD$. The rhombus is suspended from A and is prevented from collapsing by an elastic string which joins A to C, with $\angle BAD = 2\theta$, $0 \leq \theta \leq \frac{1}{3}\pi$, as shown in Fig. 2. The natural length of the elastic string is $2a$ and its modulus of elasticity is $4mg$.

(*a*) Show that the potential energy, V, of the system is given by

$$V = 4mga[(2\cos\theta - 1)^2 - 2\cos\theta] + \text{constant.}$$

(5)

(*b*) Hence find the non-zero value of θ for which the system is in equilibrium.

(4)

(*c*) Determine whether this position of equilibrium is stable or unstable.

(4)

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5. At time $t = 0$ particles P and Q start simultaneously from points which have position vectors $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ m and $(-\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ m respectively, relative to a fixed origin O. The velocities of P and Q are $(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ m s^{-1} and $(2\mathbf{i} + \mathbf{k})$ m s^{-1} respectively.

(*a*) Show that P and Q collide and find the position vector of the point at which they collide.

(5)

A third particle R moves in such a way that its velocity relative to P is parallel to the vector $(-5\mathbf{i} + 4\mathbf{j} - \mathbf{k})$ and its velocity relative to Q is parallel to the vector $(-2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$.

Given that all three particles collide simultaneously, find

(*b*) (i) the velocity of R,

(ii) the position vector of R at time $t = 0$.

(9)

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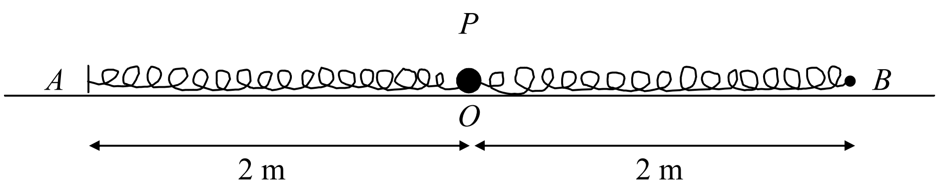
6. **Figure 3**



A particle P of mass 2 kg is attached to the mid-point of a light elastic spring of natural length 2 m and modulus of elasticity 4 N. One end A of the elastic spring is attached to a fixed point on a smooth horizontal table. The spring is then stretched until its length is 4 m and its other end B is held at a point on the table where $AB = 4$ m. At time $t = 0$, P is at rest on the table at the point O where $AO = 2$ m, as shown in Fig. 3. The end B is now moved on the table in such a way that AOB remains a straight line. At time t seconds, $AB = (4 + \frac{1}{2}\sin 4t)$ m and $AP = (2 + x)$ m.

(*a*) Show that

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4x = \sin 4t.$$

(5)

(*b*) Hence find the time when P first comes to instantaneous rest.

(12)

END