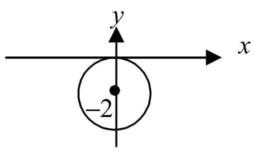
EDEXCEL PURE MATHEMATICS P6 (6676) – JUNE 2002 PROVISIONAL MARK SCHEME

Question Number	Scheme	Marks
1.	$\sinh(i\pi-\theta)=\dfrac{e^{i\pi-\theta}-e^{-(i\pi-\theta)}}{2}$	M1
	$=\dfrac{(-1)e^{-\theta}-e^{\theta}\div(-1)}{2}$	B1
	$=\dfrac{-e^{-\theta}+e^{\theta}}{2}$	A1
	$=\sinh\theta$ (✱) cso	A1
Alt.	$\sinh(i\pi-\theta)=\sinh i\pi\cosh\theta-\cosh i\pi\sinh\theta$	M1
	$=0-(-1)\sinh\theta$	B1, A1
	$=\sinh\theta$ (✱) cso	A1
		(4 marks)
2.	$x_0=0.5,\ \ y_0=1 \qquad \left(\dfrac{dy}{dx}\right)_0=0.5+e=3.2183$	B1
	$y_1=y_0+h\left(\dfrac{dy}{dx}\right)_0=1+0.1(0.5+e)=1.05+0.1e$	M1
	$=1.3218$	A1 ft
	Now $x_1=0.6,\ y_1=1.3218\ \ \therefore\left(\dfrac{dy}{dx}\right)_1=0.6+e^{1.3218}=4.3502$	M1
	and $y_2=1.3218+0.1(4.3502)=1.7568$ (or 1.7569) cao	A1
		(5 marks)
3. (*a*)(i)	$\lvert x+(y-2)i\rvert=2\lvert x+(y+i)\rvert$	M1
	$\therefore x^2+(y-2)^2=4(x^2+(y+1)^2)$	
(ii)	so $3x^2+3y^2+12y=0$ any correct from; 3 terms; isw	A1 **(2)**
	 Sketch circle	B1
	Centre (0,–2)	B1
	$r=2$ or touches axis	B1 **(3)**
(*b*)	$w=3(z-7+11i)$	B1
	$=3z-21+33i$	B1 **(2)**
		(7 marks)

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Question Number	Scheme	Marks
4. (*a*)	$y\frac{d^3y}{dx^3}+\frac{dy}{dx}\frac{d^2y}{dx^2};+2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2};+\frac{dy}{dx}=0$ marks can be awarded in (b)	M1 A1; B1;B1
	$\frac{d^3y}{dx^3}=\frac{-3\frac{dy}{dx}\frac{d^2y}{dx^2}-\frac{dy}{dx}}{y}$ or sensible correct alternative	B1 **(5)**
(*b*)	When $x=0$ $\frac{d^2y}{dx^2}=-2$, and $\frac{d^3y}{dx^3}=5$	M1A1, A1 ft
	$\therefore \quad y=1+x-x^2+\frac{5}{6}x^3...$	M1, A1 ft **(5)**
(*c*)	Could use for x = 0.2 but not for x = 50 as	B1
	approximation is best at values close to x = 0	B1 **(2)**
		(12 marks)

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Question Number	Scheme	Marks
5. (a)	$\begin{pmatrix}1&0&4\\0&5&4\\4&4&3\end{pmatrix}\begin{pmatrix}2\\-2\\1\end{pmatrix}=\begin{pmatrix}6\\-6\\3\end{pmatrix}=3\begin{pmatrix}2\\-2\\1\end{pmatrix}$, $\therefore$ eigenvalue is 3	M1A1, A1 **(3)**
(b)	Either $\begin{vmatrix}-8&0&4\\0&-4&4\\4&4&-6\end{vmatrix}=-8(24-16)+4(16)=-64+64=0$	M1 A1 **(2)**
Alt(b)	or $\begin{vmatrix}1-\lambda&0&4\\0&5-\lambda&4\\4&4&3-\lambda\end{vmatrix}=0 \Rightarrow$	
	$(1-\lambda)(5-\lambda)(3-\lambda)-16(1-\lambda)-16(5-\lambda)=0$	
	$\Rightarrow (3-\lambda)(\lambda-9)(\lambda+3)=0 \Rightarrow \lambda$ is an eigenvalue	M1 A1
	$\begin{pmatrix}x\\y\\z\end{pmatrix}$ eigenvector $\Rightarrow$ $x+4z=9x$, $5y+4z=9y$, $4x+4y+3z+9z$	M1
	At least two of these equations	
	Attempt to solve $z=2x$, $z=y$, $2x+2y=3z$	A1
	$\therefore \begin{pmatrix}1\\2\\2\end{pmatrix}$	A1 **(3)**
(c)	Make e.vectors unit to obtain $\mathbf{P}=\begin{pmatrix}\frac{2}{3}&\frac{1}{3}&\frac{2}{3}\\-\frac{2}{3}&\frac{2}{3}&\frac{1}{3}\\\frac{1}{3}&\frac{2}{3}&-\frac{2}{3}\end{pmatrix}$ columns in any order	M1, A1ft
	$\mathbf{D}=\begin{pmatrix}3&0&0\\0&9&0\\0&0&\lambda_3\end{pmatrix}$, where $\lambda_3=-3$, **P** and **D** consistent	M1, A1, B1 **(5)**
Alt	$\mathbf{P}=\begin{pmatrix}2&1&2\\-2&2&1\\1&2&-2\end{pmatrix}$, $\mathbf{D}=\begin{pmatrix}27&0&0\\0&81&0\\0&0&-27\end{pmatrix}$, **P** and **D** consistent	M1A1ft, M1A1, B1
		(13 marks)

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Question Number	Scheme	Marks
6. (*a*)	For $n = 1$ $2^5 + 5^2 = 57$, which is divisible by 3	M1, A1
	Assume true for $n = k$ $(k + 1)$th term is $2^{3k+5} + 5^{k+2}$	B1
	$(k + 1)$th term $\pm$ kth term $= 2^{3k+5} + 5^{k+2} \pm 2^{3k+2} + 5^{k+1}$	M1
	$= 2^{3k+2}(2^3 \pm 1) + 5^{k+1}(5 \pm 1)$	M1, A1
	$= 6(2^{3k+2} + 5^{k+1}) + 3.\ 2^{3k+2}$ or $= 4(2^{3k+2} + 5^{k+1}) + 3.\ 2^{3k+2}$	M1
	which is divisible by 3 $\Rightarrow$ $(k + 1)$th term is divisible by 3	A1
	Thus by induction frue for all n cso	B1 **(9)**
(*b*)	For $n = 1$ RHS $= \begin{pmatrix} -2 & -1 \\ 9 & 4 \end{pmatrix}$	B1
	Assume true for $n = k$	
	$\begin{pmatrix} -2 & -1 \\ 9 & 4 \end{pmatrix}^{k+1} = \begin{pmatrix} -2 & -1 \\ 9 & 4 \end{pmatrix}\begin{pmatrix} 1-3k & -k \\ 9k & 3k+1 \end{pmatrix} = \begin{pmatrix} -2-3k & 2k-3k-1 \\ 9+9k & -9k+12k+4 \end{pmatrix}$	M1 A3/2/1/0 (-1each error)
	$= \begin{pmatrix} 1-3(k+1) & -(k+1) \\ 9(k+1) & 3(k+1)+1 \end{pmatrix}$	B1
	$\therefore$ If true for k then true for $k + 1$ $\therefore$ by induction true for all n	B1 **(7)**
		(16 marks)

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Question Number	Scheme	Marks
7. (*a*)	$\overrightarrow{AB} = 5\mathbf{i} + 3\mathbf{j}$ $\quad \overrightarrow{AC} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ or $\overrightarrow{BC} = -2\mathbf{i} - \mathbf{j} - \mathbf{k}$	
	$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 3 & 0 \\ 3 & 2 & -1 \end{vmatrix} = -3\mathbf{i} + 5\mathbf{j} + \mathbf{k}$	M1, A1
	$\therefore \mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} + \mathbf{k})$	B1 ft **(3)**
(*b*)	Volume = $\frac{1}{6}\overrightarrow{AD}.(\overrightarrow{AB} \times \overrightarrow{AC})$ $\quad \overrightarrow{AD} = 2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$	B1
	$= {}^{1}/_{6}(2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}).(-3\mathbf{i} + 5\mathbf{j} + \mathbf{k})$	M1
	$= {}^{11}/_{6}$	A1 **(3)**
(*c*)	$\mathbf{r}.(-3\mathbf{i} + 5\mathbf{j} + \mathbf{k}) = (2\mathbf{i} + \mathbf{j}).(-3\mathbf{i} + 5\mathbf{j} + \mathbf{k})$	M1, A1 ft
	$= -1$	A1 **(3)**
(*d*)	$[\mathbf{i}.(1 - 3\lambda) + \mathbf{j}(2 + 5\lambda) + \mathbf{k}(1 + \lambda)].(-3\mathbf{i} + 5\mathbf{j} + \mathbf{k}) = -1$	M1, A1ft
	$-3 + 9\lambda + 10 + 25\lambda + 3 + \lambda = -1$	
	$35\lambda + 10 = -1 \quad \Rightarrow \lambda = -\frac{11}{35}$	M1
	$\therefore$ E is $\left(\frac{68}{35}, \frac{15}{35}, \frac{94}{35}\right)$	A1 **(4)**
(*e*)	Distance = $-\frac{11}{35}\left\|-3i + 5j + k\right\| = \frac{11\sqrt{35}}{35}$ (✱)	M1 A1 **(2)**
(*f*)	$\lambda = 2 \times \left(-\frac{11}{35}\right) = -\frac{22}{35}$	B1
	$\mathbf{r}_{D'} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + -\frac{22}{35}(-3\mathbf{i} + 5\mathbf{j} + \mathbf{k})$	M1
	D′ is $\left(\frac{101}{35}, -\frac{40}{35}, \frac{83}{35}\right)$	A1 **(3)**
		(18 marks)

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