Edexcel S2 – June 2001 – Solutions

1ai) As it is a small village, a census can be undertaken. The sampling frame would be the electoral register, or any other suitable list.

aii) For collecting traffic flow details, a sample survey will need to be used. The sampling frame would be a list of days/dates/times in which traffic could be counted.

b) The statistic could be "the number of cars passing a point in ten minutes" and this is likely to be modelled by the Poisson Distribution.

2a) Let X = "the number of accidents in the next month" Then X ~ Po(0.9)

$$P(X=0)=e^{-0.9}=0.407$$

b) Let Y = "the number of accidents in the next six months", then Y~Po(5.4)

$$P(Y=2)=\frac{e^{-5.4}(5.4)^2}{2!}=0.06659$$

c) If M = "number of months without accidents",
then from a) above M ~ Bin(4, 0.407)

$$P(M=0)=\binom{4}{2}(0.407)^0(0.593)^2=0.350$$

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3) $H_0: p = \frac{1}{4}$ $H_1: p \neq \frac{1}{4}$

We require a two tailed test of binomial proportion at the 5% level.
Assuming Ho to be correct, then X ~ Bin(20, 0.25)

From tables, critical region defined as $X \leq 1, \quad X \geq 9$

We are told exactly 2 are gold. This is not within the critical region, and so we do not reject Ho. There is insufficient evidence at this level to suggest the proportion of gold beads has changed.

4a) Let X = "t number of letters that are marked first class" then X ~ Bin(10, 0.2)

$P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.6778 = 0.3222$ (from Cum. Bin. Tables)

b) $P(X < 2) = P(X \leq 1) = 0.3758$ (from Cum. Bin. Tables)

c) Using the normal approximation, F ~ N(14, 11.2)

$$P(F \le 12) = P(F \le 12.5) = P\left(Z \le \frac{12.5 - 14}{\sqrt{11.2}}\right)$$
$$= P(Z \le -0.4482)$$
$$= 0.3264$$

d) We assume that all letters are independent of each other.

5a) Let X = "number of lightbulbs required in a week", then X ~ Po (2)

$P(X = 4) = P(X \le 4) - P(X \le 3) = 0.9473 - 0.8571 = 0.0902$ (from tables)

b) $P(X > 5) = 1 - P(X \le 4) = 1 - 0.9834 = 0.0166$ (from tables)

c) Let Y = "number of lightbulbs required in 3 weeks), then Y ~ Po(6)

$P(Y \le 5) = 0.4457$ (from tables)

d) $H_0 : \lambda = 8; \quad H_1 : \lambda < 8$

We require a one tailed test of Poisson mean at the 5% significance level.
Assuming Ho true, X ~ Po(8)

From tables, critical region is X ≤ 3

We are told 3 bulbs are requested. This is within the critical region, so we reject Ho in favour of H_1 concluding there is evidence to suggest the new measures are reducing damage.

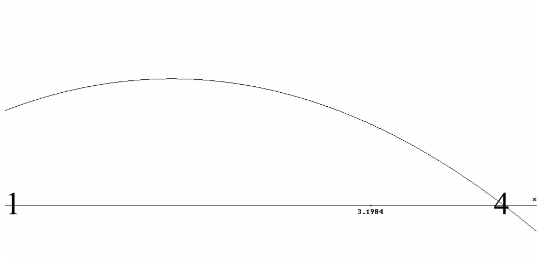
6a) pdf given by differentiation

$$f(x) = \begin{cases} \frac{1}{27}(-3x^2 + 12x) & 1 \le x \le 4 \\ 0 & \textit{otherwise} \end{cases}$$

b) Mode is given by the maximum, ie differential = 0

$-6x + 12 = 0; \quad x = 2$ Mode = 2

c)



d) $\mu = \int_1^4 x\left(\frac{1}{27}(-3x^2 + 12x\right)dx = \int_1^4 -\frac{1}{9}x^3 + \frac{4}{9}x^2 dx$

$= \left[-\frac{1}{36}x^4 + \frac{4}{27}x^3\right]_1^4$

$= \left(-\frac{256}{36} + \frac{256}{27}\right) - \left(-\frac{1}{36} + \frac{4}{27}\right)$

$= \frac{9}{4}$

e) $F(2.25) = \frac{1}{27}(-(2.25)^3 + 6(2.25)^2 - 5) = 0.517$

f) F(mean) from e) above is greater than 0.5, hence greater than the median.

$F(2) = \frac{1}{27}(-(2)^3 + 6(2)^2 - 5) = 0.407$ ie less than the median

Hence mode < median < mean

7a) $P(T < 0.2) = 0.2$

b) by symmetry $E(T) = 0.5$

c) $E(T^2) = \int_0^1 t^2 dt = \left[\frac{1}{3}t^3\right]_0^1 = \frac{1}{3}$ Hence $Var(T) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

d) Let X = "number of children stopping < 0.2" the X ~ Bin(20, 0.2)

$P(X \leq 4) = 0.6296$ (from tables)

e) I would expect the mean to remain at 0.5, but the variance to decrease as the children become more accurate.

f) $P(T < 0.2) = \int_0^{0.2} 4t\, dt = \left[2t^2\right]_0^{0.2} = 0.08$

g) Let Y = "number of players stopping < 0.2" then Y ~ Bin(75, 0.08)

As n is large, and p very small, the Poisson approximation is used. Y ~ Po(6)

$P(Y > 7) = 1 - P(Y \leq 6) = 1 - 0.7440 = 0.256$

End of Paper