UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2009**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

[Turn over

1 Solve the equation $3\tan(2x + 15°) = 4$ for $0° \leqslant x \leqslant 180°$. [4]

2 The equation of a curve is $y = 3\cos 2x$. The equation of a line is $x + 2y = \pi$. On the same diagram, sketch the curve and the line for $0 \leqslant x \leqslant \pi$. [4]

3 **(i)** Find the first 3 terms in the expansion of $(2 - x)^6$ in ascending powers of x. [3]

(ii) Given that the coefficient of x^2 in the expansion of $(1 + 2x + ax^2)(2 - x)^6$ is 48, find the value of the constant a. [3]

4 The equation of a curve is $y = x^4 + 4x + 9$.

(i) Find the coordinates of the stationary point on the curve and determine its nature. [4]

(ii) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 0$ and $x = 1$. [3]

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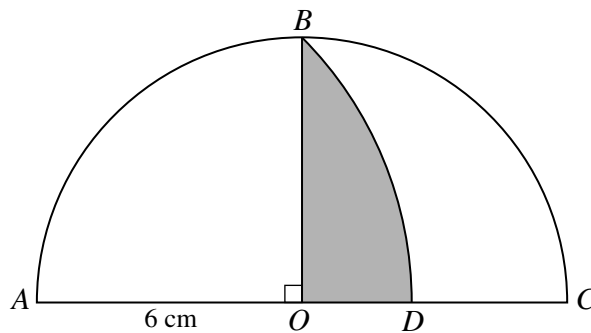
The diagram shows a semicircle ABC with centre O and radius 6 cm. The point B is such that angle BOA is $90°$ and BD is an arc of a circle with centre A. Find

(i) the length of the arc BD, [4]

(ii) the area of the shaded region. [3]

6 A curve is such that $\dfrac{dy}{dx} = k - 2x$, where k is a constant.

(i) Given that the tangents to the curve at the points where $x = 2$ and $x = 3$ are perpendicular, find the value of k. [4]

(ii) Given also that the curve passes through the point $(4, 9)$, find the equation of the curve. [3]

7 The equation of a curve is $y = \dfrac{12}{x^2 + 3}$.

    (i) Obtain an expression for $\dfrac{dy}{dx}$. [2]

    (ii) Find the equation of the normal to the curve at the point $P\,(1,\ 3)$. [3]

    (iii) A point is moving along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.012 units per second. Find the rate of change of the y-coordinate as the point passes through P. [2]

8 The first term of an arithmetic progression is 8 and the common difference is d, where $d \neq 0$. The first term, the fifth term and the eighth term of this arithmetic progression are the first term, the second term and the third term, respectively, of a geometric progression whose common ratio is r.

    (i) Write down two equations connecting d and r. Hence show that $r = \frac{3}{4}$ and find the value of d. [6]

    (ii) Find the sum to infinity of the geometric progression. [2]

    (iii) Find the sum of the first 8 terms of the arithmetic progression. [2]

9 Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 0 \\ -6 \\ 8 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} -2 \\ 5 \\ -2 \end{pmatrix}.$$

    (i) Find angle AOB. [4]

    (ii) Find the vector which is in the same direction as $\overrightarrow{AC}$ and has magnitude 30. [3]

    (iii) Find the value of the constant p for which $\overrightarrow{OA} + p\,\overrightarrow{OB}$ is perpendicular to $\overrightarrow{OC}$. [3]

10 Functions f and g are defined by

$$\text{f} : x \mapsto 2x + 1, \quad x \in \mathbb{R}, \ x > 0,$$
$$\text{g} : x \mapsto \frac{2x - 1}{x + 3}, \quad x \in \mathbb{R}, \ x \neq -3.$$

    (i) Solve the equation $\text{gf}(x) = x$. [3]

    (ii) Express $\text{f}^{-1}(x)$ and $\text{g}^{-1}(x)$ in terms of x. [4]

    (iii) Show that the equation $\text{g}^{-1}(x) = x$ has no solutions. [3]

    (iv) Sketch in a single diagram the graphs of $y = \text{f}(x)$ and $y = \text{f}^{-1}(x)$, making clear the relationship between the graphs. [3]

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