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General Certificate of Education

2016

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# Mathematics

Assessment Unit M3

*assessing*

Module M3: Mechanics 3



AMM31

[AMM31]

**TUESDAY 21 JUNE, MORNING**

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## TIME

1 hour 30 minutes.

## INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all six** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

Answers should include diagrams where appropriate and marks may be awarded for them.

Take  $g = 9.8 \text{ m s}^{-2}$ , unless specified otherwise.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

- 1 **Fig. 1** below shows two light elastic strings AP and PB used to keep a particle of weight 68 N in equilibrium at P.  
The ends A and B are attached to points 1.7 m apart on a fixed horizontal beam.  
AP is stretched to 0.8 m and BP is stretched to 1.5 m.

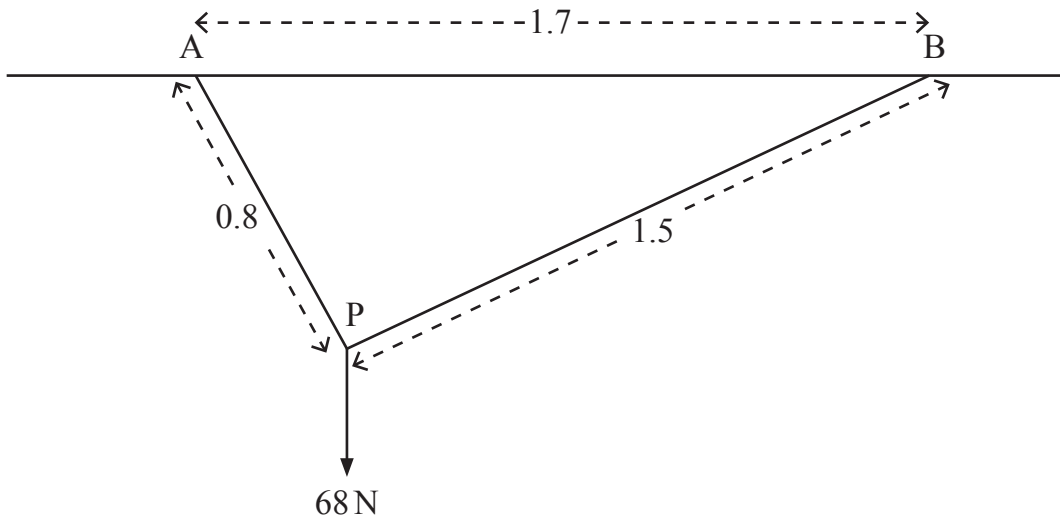


Fig. 1

- (i) By resolving in the direction of AP, or otherwise, show that the tension in AP is 60 N. [4]

The extension of AP is 0.2 m.

- (ii) Find the modulus of elasticity of AP. [2]

The stored elastic energy in BP is 8 J.

- (iii) Find the extension in BP. [6]

2 A particle P is moving along the line whose vector equation is

$$\mathbf{r} = \lambda \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$$

under the action of two constant forces,  $\mathbf{F}_1$  and  $\mathbf{F}_2$  newtons where

$$\mathbf{F}_1 = \begin{pmatrix} -4 \\ 3 \\ -2 \end{pmatrix} \text{ and } \mathbf{F}_2 = \begin{pmatrix} 2k-4 \\ 4-k \\ k-9 \end{pmatrix}$$

A and B are two points on the line where  $\lambda$  takes the values 1 and  $-2$  respectively. The distance AB is measured in metres.

(i) Show that the work done by  $\mathbf{F}_1$  as P is moved from A to B is 54 J. [5]

The mass of P is 4 kg.  
P is moving at  $1 \text{ m s}^{-1}$  at A. P is moving at  $8 \text{ m s}^{-1}$  at B.

(ii) Use the Work–Energy Principle to find  $k$ . [5]

3 The equation of motion of a particle moving in a straight line with S.H.M. of amplitude  $a$  is given by

$$\ddot{x} = -\omega^2 x$$

where  $x$  is the displacement from the centre of oscillation.

A particle P moving with S.H.M., of period  $4\pi$  seconds, has velocity  $1.4 \text{ m s}^{-1}$  when  $x$  is 4.5 m.

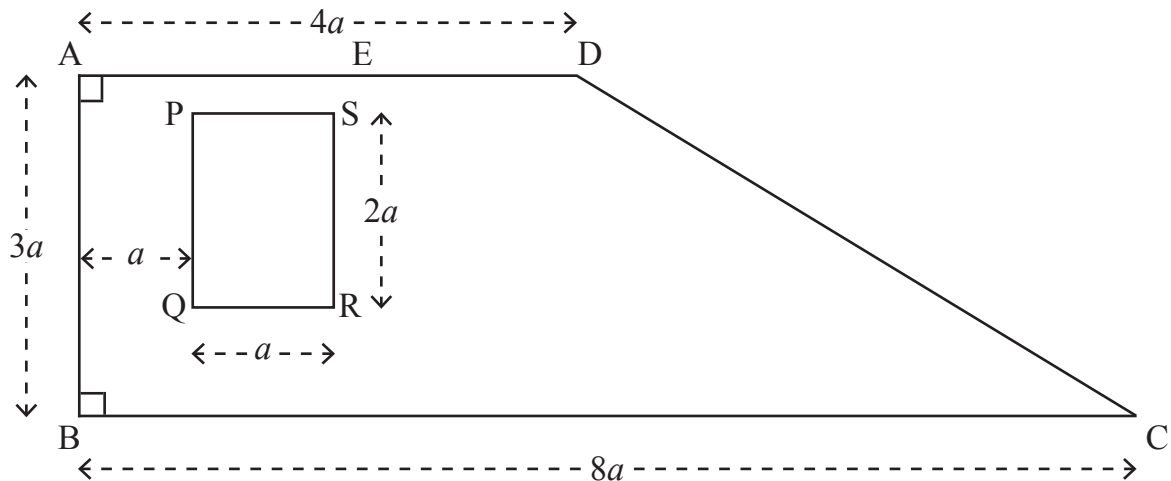
(i) Find  $\omega$  and  $a$ . [4]

(ii) Find the maximum speed of P during the motion. [1]

(iii) Find the maximum magnitude of the acceleration of P during the motion. [1]

(iv) Find the total time, in one complete oscillation, for which the speed of P is less than or equal to  $1.4 \text{ m s}^{-1}$  [5]

- 4 **Fig. 2** below shows a design for a uniform metal logo ABCD in the shape of a trapezium with a rectangle PQRS removed.



**Fig. 2**

$$\hat{A}BC = \hat{D}AB = 90^\circ$$

$AB = 3a$  cm,  $AD = 4a$  cm and  $BC = 8a$  cm.

PQ is parallel to AB.

The distance between the parallel sides PQ and AB is  $a$  cm.

$PQ = 2a$  cm and  $QR = a$  cm.

Model the logo as a lamina.

The centre of mass of the logo is at G.

- (i) Show that the distance of G from AB is  $\frac{53a}{16}$  cm. [6]

E is the point on AD such that  $AE = 2.5a$  cm.

The mass of the removed metal rectangle PQRS is  $m$  kg.

A small metal stud of mass  $M$  kg is attached at A.

- (ii) When the logo is freely suspended from E, AD is horizontal.  
Find  $M$  in terms of  $m$ . [4]

- (iii) If, instead, the logo is freely suspended from A, find the minimum force in terms of  $m$  and  $g$  required to keep AD horizontal. [3]

- 5 O is a fixed point on a straight horizontal line. A particle P of mass 6 kg is a distance  $x$  metres from O, where  $x \geq 0$

A variable force  $F$  newtons acts on P in the direction OP.  $F$  is given by

$$F = 10 + 0.5e^{\frac{x}{2}} - 2x$$

- (i) Find the acceleration of P at O. [2]

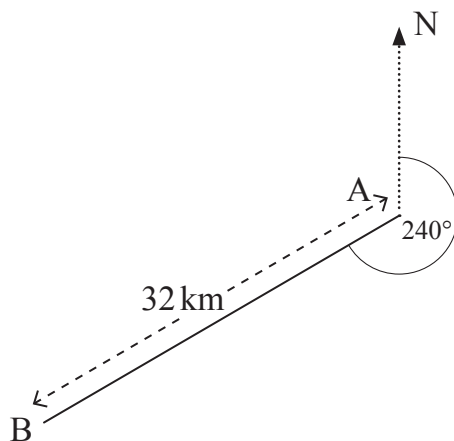
- (ii) Show that the work done by  $F$  for  $0 \leq x \leq 6$  is approximately 43.1 J [5]

The speed of P at O is  $5 \text{ m s}^{-1}$

- (iii) Find the speed of P when  $x = 6$  [2]

- (iv) Find  $x$  when  $F$  is a minimum. [6]

- 6 A ship B is 32 km away from a ship A on a bearing of  $240^\circ$  as shown in **Fig. 3** below.

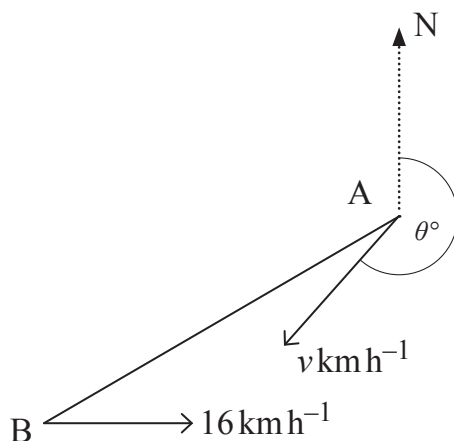


**Fig. 3**

Ship B is sailing due east at  $16 \text{ km h}^{-1}$

A sets out to intercept B.

A travels at  $v \text{ km h}^{-1}$  on a bearing of  $\theta^\circ$  as shown in **Fig. 4** below.



**Fig. 4**

- (i) When  $v = 24$ , show that  $\theta^\circ$  is approximately  $221^\circ$  [5]
- (ii) Find the least value of  $v$  for which A can intercept B. [1]
- (iii) When  $v = 12$ , show, by drawing a velocity diagram, that A has a choice of two courses to intercept B. Hence, find the greater time taken to intercept. [8]

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**THIS IS THE END OF THE QUESTION PAPER**

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