Rewarding Learning

ADVANCED
General Certificate of Education
2016

Mathematics

Assessment Unit M2
assessing
Module M2: Mechanics 2

AMM21

[AMM21]
THURSDAY 2 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A particle, P, of mass 4 kg is acted on by a force $(-2\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$ N.
Initially, P is at rest at a point A.
After 6 seconds, P reaches the point B.

(i) Find the velocity of P at B. [4]

A is $(10\mathbf{i} + 12\mathbf{k})$ m from a fixed point O.

(ii) Find the displacement vector $\overrightarrow{OB}$. [4]

(iii) Find the unit vector in the direction of P's motion. [4]

2 **Fig. 1** below, shows a cricket ball being hit from a height of 0.9 m above horizontal ground with a speed of 25 $\mathrm{m\,s^{-1}}$ at an angle of θ above the horizontal, where $\tan\theta = \frac{7}{24}$

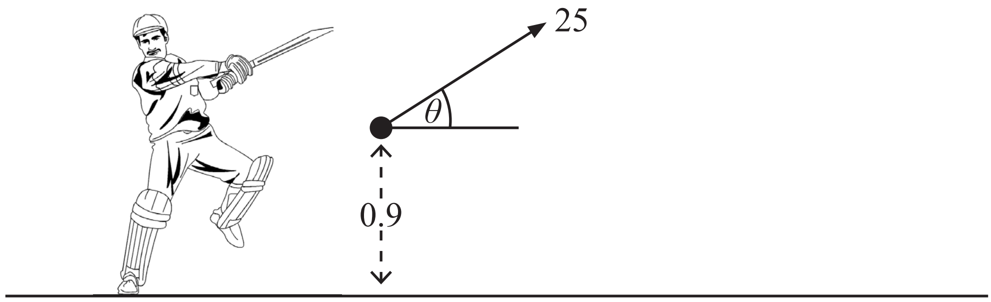

Fig. 1

The motion of the cricket ball can be modelled as that of a particle moving freely under gravity.

(i) Find the length of time for which the ball is at least 3 m above the ground. [5]

The ball is caught by a fielder who is 33 m horizontally from the point where it was struck.

(ii) Find the vertical height above the ground, of the ball when it is caught. [5]

(iii) State, briefly, one way in which the above model could be refined in order to make it more realistic. [1]

3 A particle P moves so that at time t seconds, its velocity $\mathbf{v}$ is given by

$$\mathbf{v} = \left[(3t^2 - 6)\,\mathbf{i} - 6t^2\,\mathbf{j}\right]\,\mathrm{m\,s^{-1}}$$

At time $t = 0$, the displacement of P from a fixed point O is $(15\mathbf{i} + 75\mathbf{j})\,\mathrm{m}$.

(i) Find the position vector of P from O at time t. [4]

The position vector of a second particle Q from O at the same time t is given by

$$\overrightarrow{\mathrm{OQ}} = \left[(t^3 - 3t)\,\mathbf{i} + (3t^2 - 2t^3)\,\mathbf{j}\right]\,\mathrm{m}$$

(ii) Find the value of t when P and Q meet. [4]

(iii) Find the velocity of Q when the particles meet. [3]

4 **Fig. 2** below shows two particles P and Q of mass 3 kg and m kg respectively, connected by a light inextensible string of length 6 m.
The string passes through a small, smooth ring fixed at O.
P hangs in equilibrium 4 m vertically below O and Q moves with a constant speed in a horizontal circle about OP.
The string OQ makes an angle of θ with the horizontal, where $\theta = \sin^{-1}\left(\frac{2}{5}\right)$

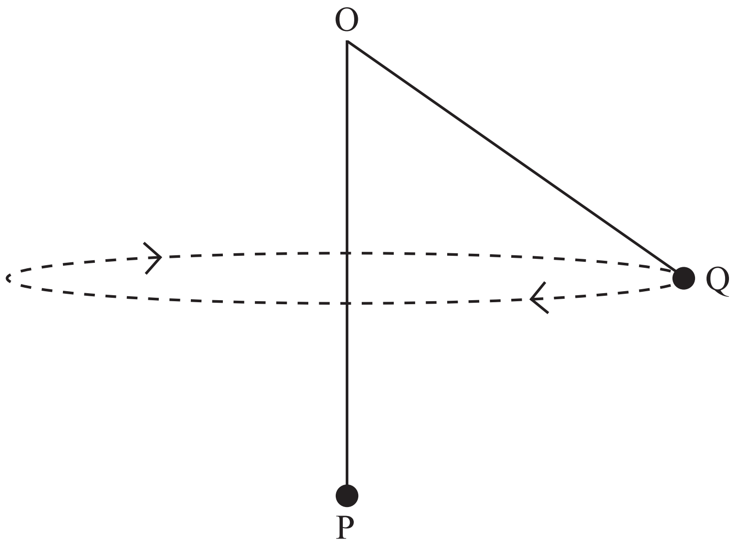

Fig. 2

(i) Draw a diagram showing the external forces acting on P and Q. [2]

(ii) Find m. [4]

Q rotates with an angular velocity of ω rad s^{-1}

(iii) Find ω. [5]

5 A van is travelling up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{12}$
The resistances to the motion of the van can be modelled as a constant force of 1.2 kN.
Initially, the speed of the van is $30\,\mathrm{m\,s^{-1}}$ and the engine of the van is working at a rate of 60 kW. At this point, the van is decelerating at $0.3\,\mathrm{m\,s^{-2}}$

(i) Show that the mass of the van is 1548 kg. [6]

While travelling up the same road, the rate of working of the van's engine is now increased to 78 kW. Assume resistances remain as before.

(ii) Find the maximum speed of the van at this rate of working. [4]

(iii) State why in a more realistic model, the resistances to motion would not be constant. [1]

6 A pump draws water from a tank and issues it from the end of the hose which is 8 m vertically above the level from which the water is drawn.
The cross-sectional area of the hose is $A\,\mathrm{m^2}$ and the water leaves the end of the hose at a speed of $16\,\mathrm{m\,s^{-1}}$
The pump works at a rate of 500 W.
The density of water is $1000\ \mathrm{kg\,m^{-3}}$

Find A. [7]

7 At time $t = 0$ seconds, a parachutist of mass m kg jumps from rest from the top of a cliff and falls freely under gravity.
The air resistance, at speed $v\,\mathrm{m\,s^{-1}}$, can be modelled as kmv newtons, where k is a constant.
Model the parachutist as a particle.
The terminal velocity of the parachutist, $V\,\mathrm{m\,s^{-1}}$, is her theoretical maximum speed during her fall.

(i) Show that $V = \frac{g}{k}$ [3]

(ii) Find an expression, in terms of k, for the time taken for her to reach a speed of $\frac{V}{2}$ [9]

THIS IS THE END OF THE QUESTION PAPER