Rewarding Learning

ADVANCED
General Certificate of Education
2015

Mathematics

Assessment Unit M2

assessing

Module M2: Mechanics 2

AMM21

[AMM21]

MONDAY 22 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A particle P moves so that at time t seconds, its displacement **s** metres from a fixed point O is given by

$$\mathbf{s} = (t^2 + 2)\mathbf{i} + (t^3 - 1)\mathbf{j} + (t^2 + 2t)\mathbf{k}$$

(i) Show that P starts its motion on the **i**–**j** plane. [2]

(ii) Find the speed of P at $t = 2$ [4]

(iii) Find t when the acceleration of the particle is $(2\mathbf{i} + 8\mathbf{j} + 2\mathbf{k})\ \mathrm{m\,s^{-2}}$ [3]

2 A particle, P, of mass 3 kg is acted upon by two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$

$\mathbf{F}_1$ has magnitude 18 N and acts in the direction of the vector $\begin{pmatrix} 4 \\ -4 \\ -7 \end{pmatrix}$

$\mathbf{F}_2$ has magnitude 6 N and acts in the direction of the vector $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

(i) Find, in vector form, the resultant force acting on P. [4]

(ii) Hence, show that the acceleration of P is $\begin{pmatrix} 4 \\ -2 \\ -6 \end{pmatrix}\ \mathrm{m\,s^{-2}}$ [2]

At time $t = 0$ seconds, P is moving with velocity $\begin{pmatrix} -1 \\ 3 \\ 6 \end{pmatrix}\ \mathrm{m\,s^{-1}}$

(iii) Find the velocity of P at $t = 1$ [2]

(iv) Show that at $t = 1$, the particle is moving at right angles to its initial direction. [2]

3 **Fig. 1** below shows a cyclist, Sharon, moving along a cycle track ABC.
B is the lowest point on the track.
A is 60m vertically above the horizontal through B.
C is 25m vertically above the horizontal through B.

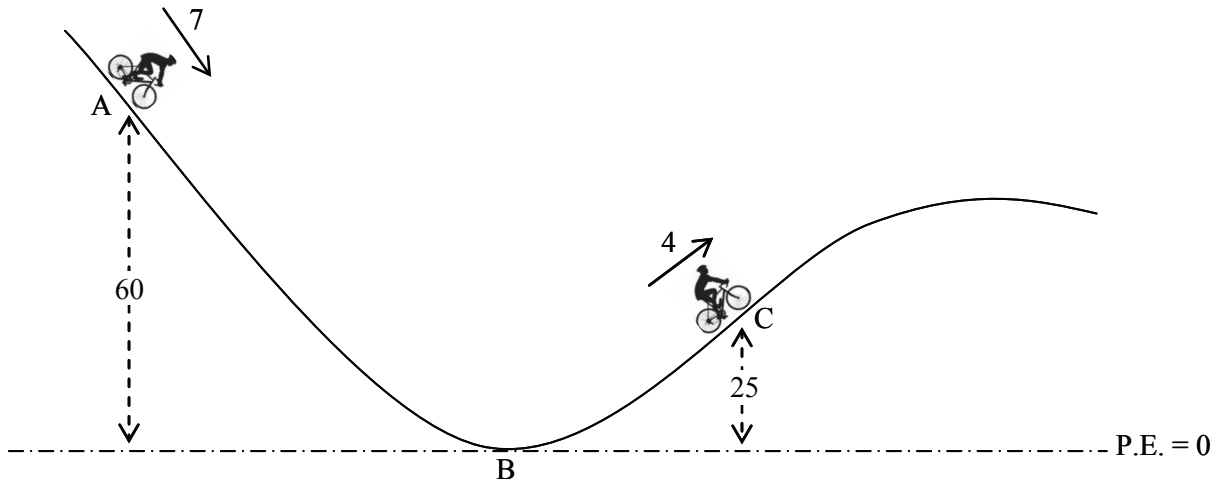

Fig. 1

Sharon and her cycle have a combined mass of 75 kg.
The total distance she travels along the track from A to C is 2100 m.
The total resistance to her motion is constant and is 14 N.
At A, Sharon is moving downhill at $7\,m\,s^{-1}$
At C, Sharon is moving uphill at $4\,m\,s^{-1}$
Take the potential energy at B to be zero.

(i) Find the change in the kinetic energy of the cyclist between A and C. [2]

(ii) Find the work done against resistance between A and C. [2]

(iii) Find the work done by the cyclist between A and C. [5]

4 In this question take $g = 10\,m\,s^{-2}$

A van of mass 2 tonnes has an engine which can produce a maximum power of 48 kW. The total resistance to the motion of the van can be modelled by $(a + bv)$ newtons, where a and b are constants and v is the speed of the van in $m\,s^{-1}$

The maximum speed of the van on a straight horizontal road is $50\,m\,s^{-1}$

(i) Find an equation connecting a and b. [4]

The maximum speed of the van up a hill inclined at $\sin^{-1}\left(\frac{1}{25}\right)$ to the horizontal is $30\,m\,s^{-1}$

(ii) Find a second equation connecting a and b and hence show $a = 560$ and $b = 8$ [5]

(iii) Find the acceleration of the van when it is travelling at $40\,m\,s^{-1}$ on a straight horizontal road. [3]

5 A plane is travelling horizontally at $49\,m\,s^{-1}$ at a height of 100 m vertically above the ground, as shown in **Fig. 2** below.
The plane drops a food package to people on the ground.

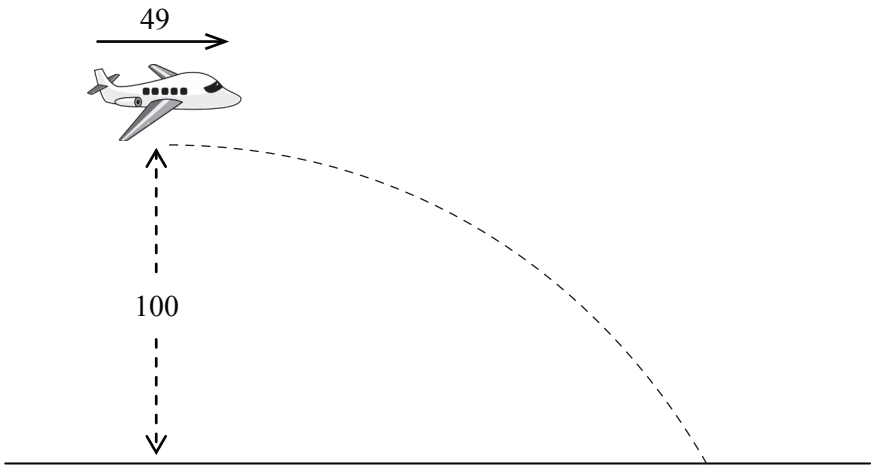

Fig. 2

Model the package as a particle.

(i) Find the horizontal distance travelled by the package just before it hits the ground. [4]

(ii) Find the speed at which the package hits the ground. [5]

(iii) State one assumption you have made in answering this question. [1]

6 A ship of mass M kg is sailing in a straight line between two ports.
When the speed of the ship is v m s^{-1}, the total resistance to its motion can be modelled by $(v^2 + k^2)$ newtons, where k is a constant.
The engines of the ship exert a constant tractive force of $5k^2$ newtons.

(i) Show that the maximum speed of the ship is $2k$ m s^{-1} [2]

As the ship passes a point P at its maximum speed, the engines are suddenly put into maximum reverse thrust of $5k^2$ N.
The total resistance to motion remains as before.

(ii) Show that the motion of the ship can be modelled by the differential equation

$$v^2 + 6k^2 = -Mv\frac{\mathrm{d}v}{\mathrm{d}x}$$

where x is the distance, in metres, after passing P. [3]

(iii) Show that the ship will momentarily stop when $x = \frac{M}{2}\ln\frac{5}{3}$ [8]

7 **Fig. 3** below shows a bead B, of mass m kg, fastened to a point on a light inextensible string of length $7x$ metres.
The ends of the string are fastened to fixed points A and C, A being vertically above C.
$\angle$ ABC = 90°, AB = $4x$ and BC = $3x$.

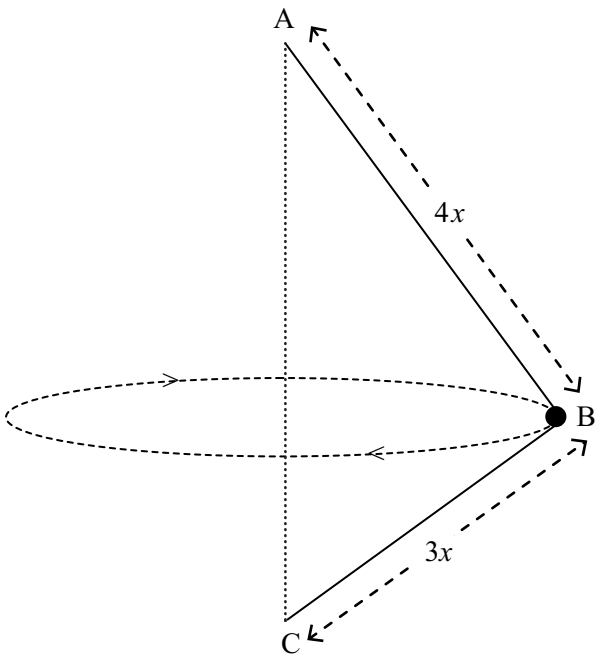

Fig. 3

B travels in a horizontal circle with constant angular velocity ω rad s^{-1}, where $\omega = \sqrt{\frac{5g}{6x}}$
The string is taut.

(i) Draw a diagram showing the external forces acting on B. [1]

(ii) Show that B travels in a circular path of radius $\frac{12x}{5}$ metres. [2]

(iii) Find, in terms of m and g, the tensions in the two parts of the string. [9]

THIS IS THE END OF THE QUESTION PAPER