Rewarding Learning

ADVANCED
General Certificate of Education
2015

Mathematics

Assessment Unit F3
assessing
Module FP3: Further Pure Mathematics 3

AMF31

[AMF31]
FRIDAY 5 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find the angle between the vector

$$8\mathbf{i} - \mathbf{j} + 3\mathbf{k}$$

and the normal to the plane

$$5x + 2y - 7z = 17$$ [4]

2 Find

$$\int \frac{\mathrm{d}x}{5 - 4x + 4x^2}$$ [4]

3 Differentiate

$$\tan^{-1}\left(\frac{\sqrt{1 - x^2}}{x - 3}\right)$$ [6]

4 **(a)** Find the intersection of the plane

$$2x - 6y + 5z = 24$$

and the line

$$\frac{x-12}{4} = \frac{y-5}{3} = \frac{z+1}{-5}$$ [4]

(b) Find the volume of the tetrahedron with vertices A (2, 1, 1), B (1, 4, 5), C (0, 2, 3) and D (5, 3, 7). [6]

5 **(i)** Using the exponential definitions of $\sinh x$ and $\cosh x$, show that

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$ [4]

(ii) If

$$\sinh x - 2\coth y = 1$$

and

$$2\sinh x - 5\coth y = -1$$

find x and y, giving your answers in logarithmic form. [6]

6 **(i)** Using the exponential definitions of $\sinh x$ and $\cosh x$, prove the identity

$$\sinh^2 x \equiv \cosh^2 x - 1$$ [3]

Let $I_n = \int \cosh^n x \, dx \quad n \geqslant 0$

(ii) Show that

$$I_n = \frac{1}{n}\cosh^{(n-1)} x \sinh x + \frac{n-1}{n} I_{n-2} \qquad n \geqslant 2$$ [8]

(iii) Find the exact value of

$$\int_0^{\ln 2} \cosh^4 x(1 - \sinh x)\,dx$$ [9]

7 The pyramid ABCDE has the irregular quadrilateral ABCD for its base as shown in **Fig. 1** below.

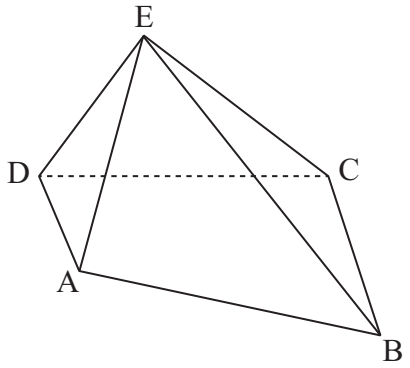

Fig. 1

The line AB is given by $\{\mathbf{r} - (-6\mathbf{i} - 4\mathbf{j} - 2\mathbf{k})\} \times (10\mathbf{i} - 2\mathbf{j} - \mathbf{k}) = \mathbf{0}$
The line BC is given by $\{\mathbf{r} - (4\mathbf{i} - 6\mathbf{j} - 3\mathbf{k})\} \times (4\mathbf{i} + 10\mathbf{j} + 5\mathbf{k}) = \mathbf{0}$
The plane CDE is given by $10x + 18y + 19z = 190$
A, B, C and D lie in the same plane.

Find a vector equation of the line CD. [9]

8 Use integration by parts to find the volume obtained by rotating the curve

$$y = \sin^{-1} x$$

through 2π radians about the x-axis between the ordinates $x = 0$ and $x = \frac{1}{2}$ [12]

THIS IS THE END OF THE QUESTION PAPER