



Rewarding Learning

ADVANCED
General Certificate of Education
2015

Mathematics

Assessment Unit M2
assessing
Module M2: Mechanics 2

MV18

[AMM21]

MONDAY 22 JUNE, AFTERNOON

TIME

1 hour 30 minutes, plus your additional time allowance.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all seven** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed at the end of each question indicate the marks awarded to each question or part question.

Answers should include diagrams where appropriate and marks may be awarded for them.

Take $g = 9.8 \text{ m s}^{-2}$, unless specified otherwise.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A particle P moves so that at time t seconds, its displacement \mathbf{s} metres from a fixed point O is given by

$$\mathbf{s} = (t^2 + 2)\mathbf{i} + (t^3 - 1)\mathbf{j} + (t^2 + 2t)\mathbf{k}$$

(i) Show that P starts its motion on the \mathbf{i} - \mathbf{j} plane. [2 marks]

(ii) Find the speed of P at $t = 2$ [4 marks]

(iii) Find t when the acceleration of the particle is $(2\mathbf{i} + 8\mathbf{j} + 2\mathbf{k}) \text{ m s}^{-2}$ [3 marks]

- 2 A particle, P, of mass 3 kg is acted upon by two constant forces \mathbf{F}_1 and \mathbf{F}_2

\mathbf{F}_1 has magnitude 18 N and acts in the direction of the vector $\begin{pmatrix} 4 \\ -4 \\ -7 \end{pmatrix}$

\mathbf{F}_2 has magnitude 6 N and acts in the direction of the vector $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

- (i) Find, in vector form, the resultant force acting on P. [4 marks]

- (ii) Hence, show that the acceleration of P is $\begin{pmatrix} 4 \\ -2 \\ -6 \end{pmatrix} \text{ m s}^{-2}$ [2 marks]

At time $t = 0$ seconds, P is moving with velocity $\begin{pmatrix} -1 \\ 3 \\ 6 \end{pmatrix} \text{ m s}^{-1}$

- (iii) Find the velocity of P at $t = 1$ [2 marks]

- (iv) Show that at $t = 1$, the particle is moving at right angles to its initial direction. [2 marks]

- 3 **Fig. 1** below shows a cyclist, Sharon, moving along a cycle track ABC.
 B is the lowest point on the track.
 A is 60 m vertically above the horizontal through B.
 C is 25 m vertically above the horizontal through B.

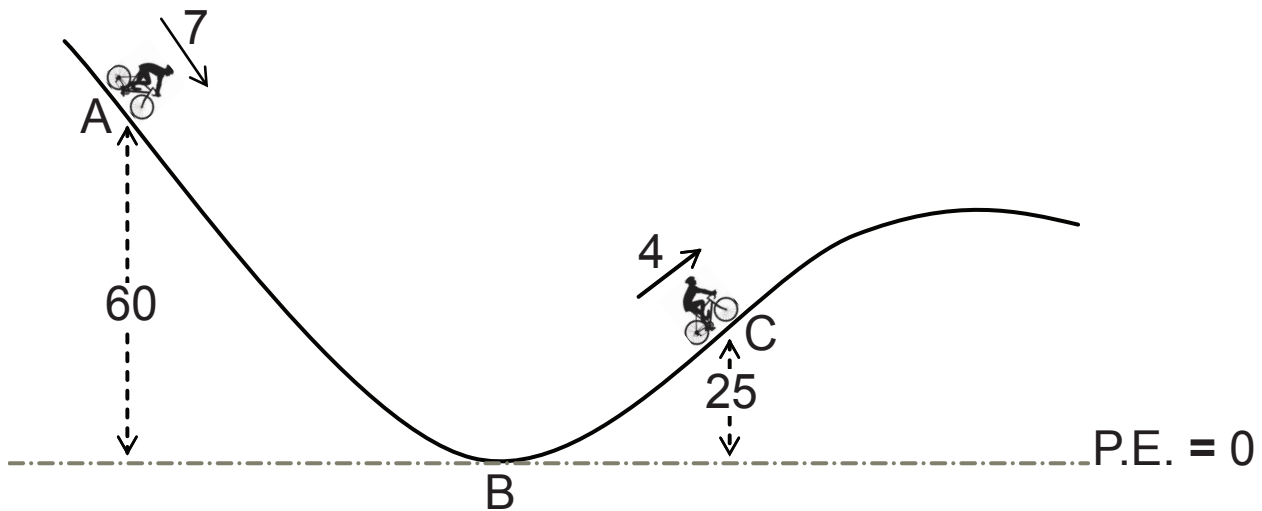


Fig. 1

Sharon and her cycle have a combined mass of 75 kg.
 The total distance she travels along the track from A to C is 2100 m.
 The total resistance to her motion is constant and is 14 N.
 At A, Sharon is moving downhill at 7 m s^{-1}
 At C, Sharon is moving uphill at 4 m s^{-1}
 Take the potential energy at B to be zero.

- (i) Find the change in the kinetic energy of the cyclist between A and C. [2 marks]
- (ii) Find the work done against resistance between A and C. [2 marks]
- (iii) Find the work done by the cyclist between A and C. [5 marks]

4 In this question take $g = 10 \text{ m s}^{-2}$

A van of mass 2 tonnes has an engine which can produce a maximum power of 48 kW.

The total resistance to the motion of the van can be modelled by $(a + bv)$ newtons, where a and b are constants and v is the speed of the van in m s^{-1}

The maximum speed of the van on a straight horizontal road is 50 m s^{-1}

(i) Find an equation connecting a and b . [4 marks]

The maximum speed of the van up a hill inclined at $\sin^{-1} \left(\frac{1}{25} \right)$ to the horizontal is 30 m s^{-1}

(ii) Find a second equation connecting a and b and hence show $a = 560$ and $b = 8$ [5 marks]

(iii) Find the acceleration of the van when it is travelling at 40 m s^{-1} on a straight horizontal road. [3 marks]

- 5 A plane is travelling horizontally at 49 m s^{-1} at a height of 100 m vertically above the ground, as shown in **Fig. 2** below.
The plane drops a food package to people on the ground.

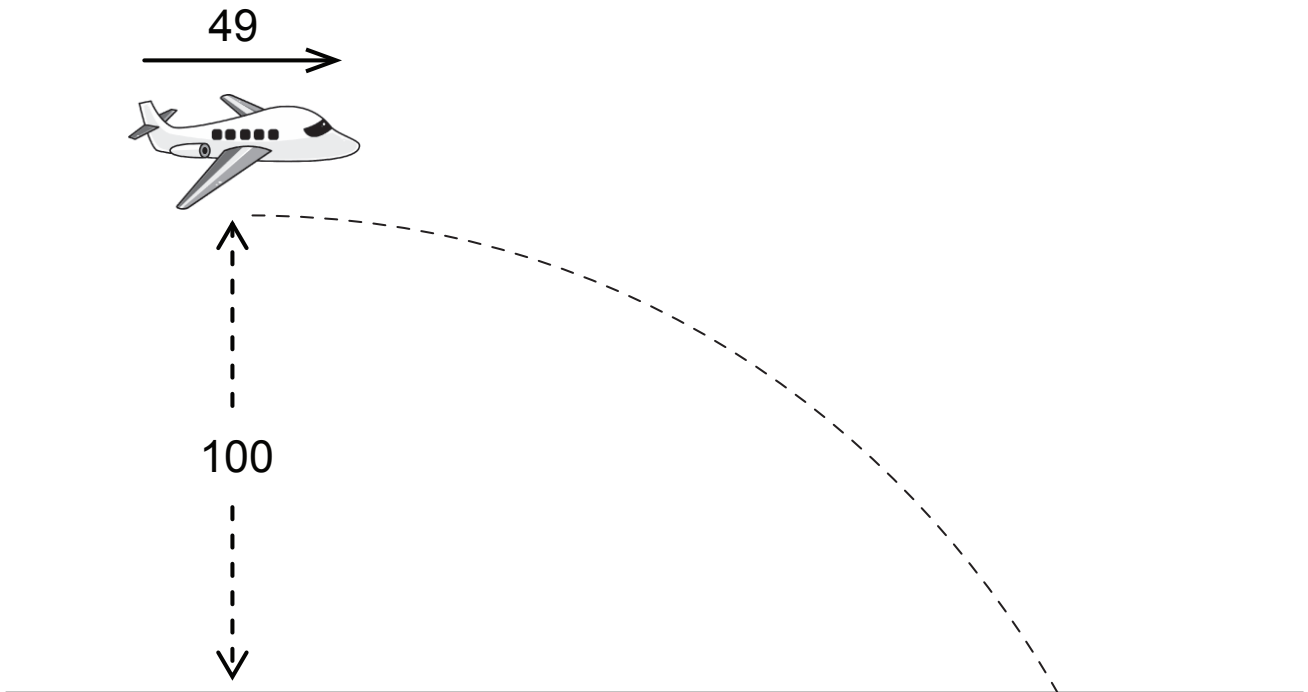


Fig. 2

Model the package as a particle.

- (i) Find the horizontal distance travelled by the package just before it hits the ground. [4 marks]
- (ii) Find the speed at which the package hits the ground. [5 marks]
- (iii) State one assumption you have made in answering this question. [1 mark]

6 A ship of mass M kg is sailing in a straight line between two ports.

When the speed of the ship is v m s^{-1} , the total resistance to its motion can be modelled by $(v^2 + k^2)$ newtons, where k is a constant.

The engines of the ship exert a constant tractive force of $5k^2$ newtons.

(i) Show that the maximum speed of the ship is $2k$ m s^{-1}
[2 marks]

As the ship passes a point P at its maximum speed, the engines are suddenly put into maximum reverse thrust of $5k^2$ N.

The total resistance to motion remains as before.

(ii) Show that the motion of the ship can be modelled by the differential equation

$$v^2 + 6k^2 = -Mv \frac{dv}{dx}$$

where x is the distance, in metres, after passing P .

[3 marks]

(iii) Show that the ship will momentarily stop when

$$x = \frac{M}{2} \ln \frac{5}{3} \quad [8 \text{ marks}]$$

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(Questions continue overleaf)

- 7 **Fig. 3** below shows a bead B, of mass m kg, fastened to a point on a light inextensible string of length $7x$ metres. The ends of the string are fastened to fixed points A and C, A being vertically above C. $\angle ABC = 90^\circ$, $AB = 4x$ and $BC = 3x$.

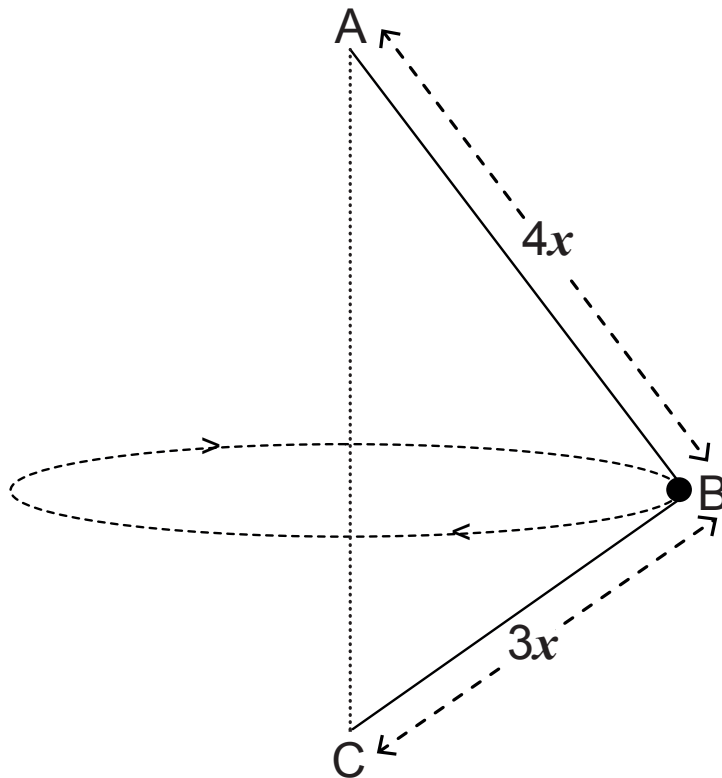


Fig. 3

B travels in a horizontal circle with constant angular velocity ω rad s^{-1} , where $\omega = \sqrt{\frac{5g}{6x}}$. The string is taut.

- (i) Draw a diagram showing the external forces acting on B. [1 mark]
- (ii) Show that B travels in a circular path of radius $\frac{12x}{5}$ metres. [2 marks]
- (iii) Find, in terms of m and g , the tensions in the two parts of the string. [9 marks]

THIS IS THE END OF THE QUESTION PAPER
