Rewarding Learning

ADVANCED
General Certificate of Education
January 2012

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]

AMF21

WEDNESDAY 1 FEBRUARY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(i)** Show that the sum of the series

$$\frac{5}{1\times2\times3}+\frac{8}{2\times3\times4}+\frac{11}{3\times4\times5}+\dots+\frac{3n+2}{n(n+1)(n+2)}$$

is given by

$$2-\frac{1}{n+1}-\frac{2}{n+2}$$ [8]

(ii) Hence find the sum of the series

$$\frac{5}{1\times2\times3}+\frac{8}{2\times3\times4}+\frac{11}{3\times4\times5}+\dots$$ [1]

2 Find the first 2 non-zero terms of the Maclaurin series for $\tan^{-1}(x)$. [6]

3 **(a)** Find the equation of an ellipse with focus F (4, 0) and associated directrix $x = 6.25$ [6]

(b) Find the equation of the parabola with focus F (4, 1) and directrix $x = -1$ [4]

4 Find the general solution of the equation

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+3y=3\cos 2x$$ [12]

5 **(i)** Using partial fractions show that

$$\frac{3}{(1+2x^2)(1-2x)} \equiv \frac{2x+1}{1+2x^2} + \frac{2}{1-2x}$$ [6]

(ii) Hence find a series expansion for

$$\frac{3}{(1+2x^2)(1-2x)}$$

up to and including the term in x^4 [5]

(iii) State the range of values of x for which the expansion is valid. [3]

6 Use the principle of mathematical induction to show that for all non-negative integers n

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n}\left[\mathrm{e}^x \cos\sqrt{3}x\right] = 2^n \mathrm{e}^x \cos\left(\sqrt{3}x + \frac{n\pi}{3}\right)$$ [8]

$\left[\text{Note that } \frac{\mathrm{d}^0}{\mathrm{d}x^0}\mathrm{f}(x) = \mathrm{f}(x)\right]$

7 **(i)** If $Z = e^{i\theta}$ show that

$$\frac{1}{2}\left(Z^n + \frac{1}{Z^n}\right) \equiv \cos n\theta \qquad [3]$$

(ii) Given that

$$\cos^6 \theta \equiv a\cos 6\theta + b\cos 4\theta + c\cos 2\theta + d$$

find a, b, c and d. [6]

(iii) Hence, find, in radians, the general solution to the equation

$$\frac{1}{8}\cos 6\theta + \frac{3}{4}\cos 4\theta + \frac{15}{8}\cos 2\theta + \frac{5}{4} = \frac{1}{2} \qquad [7]$$

THIS IS THE END OF THE QUESTION PAPER