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ADVANCED
General Certificate of Education
January 2012

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

FRIDAY 27 JANUARY, MORNING

AMC41

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Use the substitution $u = 3x + 2$ to find

$$\int (3x+2)^5 \, dx$$ [5]

2 **(a)** Find the distance between the points A $(2, -1, 3)$ and B $(-2, 2, -1)$. [2]

(b) Find the angle between the lines

$$\mathbf{r}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \qquad \text{and} \quad \mathbf{r}_2 = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$ [6]

3 A particle is moving in a straight line in such a way that its distance d metres from a fixed point O, t seconds after the motion begins, is given by

$$d = 15 \sin t + 20 \cos t \qquad 0 \leqslant t \leqslant 2\pi$$

(i) Express d in the form

$$r \sin (t + \alpha)$$

where r is a positive integer and $0 < \alpha < \frac{\pi}{2}$ [3]

(ii) Hence find the maximum distance of the particle from O and the time at which it first occurs. [4]

4 **(i)** Sketch the function $f(x) = x^2 - 2$ where $x \geqslant 0$ [2]

(ii) Hence state the range of $f(x) = x^2 - 2$ where $x \geqslant 0$ [1]

(iii) Find the inverse function $f^{-1}(x)$ and state its domain. [4]

5 **(i)** If

$$x = 3 \sin \theta \qquad \text{and} \qquad y = 2 \cos \theta$$

find $\dfrac{dy}{dx}$ [3]

(ii) Find the equation of the normal to the curve given parametrically by the equations

$$x = 3 \sin \theta \qquad \text{and} \qquad y = 2 \cos \theta$$

at the point with parameter $\theta = \dfrac{\pi}{4}$ [6]

6 Water is draining from a storage tank.
The rate of change of the depth D of water is proportional to the square of the depth at time t.

(i) Model this by a differential equation. [2]

The initial depth of water is 2 metres and after 5 minutes the depth has reduced to 1.5 m.

(ii) By solving the differential equation, find the time taken for the water to reduce to a depth of 0.8 m. [9]

7 **(a)** Sketch the graph of

$$y = \operatorname{cosec} x \qquad 0° \leqslant x \leqslant 360° \qquad [2]$$

(b) Solve the equation

$$\tan 2\theta = 4 \tan \theta \qquad 0° \leqslant \theta \leqslant 360° \qquad [9]$$

8 **(a)** **(i)** Rewrite $\dfrac{x+3}{4-x^2}$ as partial fractions. [6]

(ii) Hence find

$$\int \frac{x+3}{4-x^2}\,\mathrm{d}x \qquad [3]$$

(b) The graph of part of the curve $y = \sec 2x \tan 2x$ is shown in **Fig. 1** below.

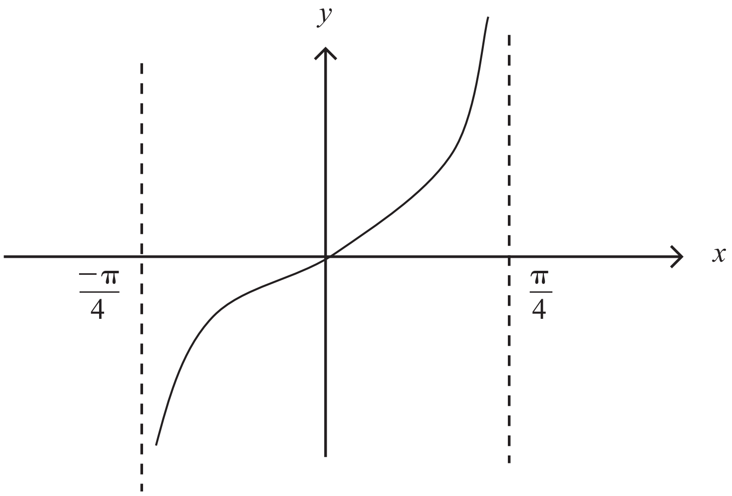

Fig. 1

Find the **exact** volume of the solid formed when the area between the curve

$$y = \sec 2x \tan 2x$$

and the x-axis between $x = 0$ and $x = \dfrac{\pi}{6}$ is rotated through 2π radians about the x-axis. [8]